Estimation of joint nonlinear continuous-discrete models and applications for the valuation of time

Dissertation Submitted

by

Simona Jokubauskaitė

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Institute of Statistics

Supervisor:

Univ.-Prof. Dipl.-Ing. Dr.techn. Friedrich Leisch

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Abstract Theoretical models are often nonlinear and, thus, the popular linearity assumption has to be relaxed. Monetary valuation of time and estimation of nonlinear models are the core of this thesis. In the joint time-use and discrete choice model (JTUDC, Munizaga et al., 2008), endogenous variables have a nonlinear functional form. Combined they make up a system of nonlinear equations. The offshoot of this model is a monetary valuation of time assigned to work, leisure, and travel. This thesis uses an innovative panel data-set, in contrast, the original model was calibrated on the cross-sectional data. The estimation procedure was advanced to account for the repeated observations via a normal error component model with a latent variable in the hierarchical Bayesian framework. Additionally, *R* package *nmm* was developed to make the estimation of the joint continuous-discrete models available to the wider public. Previously mentioned time indicators are widely used in the cost-benefit analysis of transport infrastructure projects. Due to the different time assignment patterns, the originally estimated average value of leisure for males is double the amount for females. To avoid gender-driven bias, it was proposed to treat domestic work as official work by assigning to it the market wage rate. After this adjustment, the gap vanishes almost completely.

Zusammenfassung Theoretische Modelle sind oft nichtlinear und daher muss die populäre Linearitätsannahme gelockert werden. Die monetäre Bewertung der Zeit und die Schätzung nichtlinearer Modelle bilden den Kern dieser Arbeit. Im gemeinsamen Zeitnutzungs- und diskreten Auswahlmodell (JTUDC, Munizaga et al., 2008) haben endogene Variablen eine nichtlineare funktionale Form. Zusammen bilden sie ein System nichtlinearer Gleichungen. Die Folge dieses Modells ist eine monetäre Bewertung der Zeit, die für Arbeit, Freizeit und Reisen verwendet wird. Diese Arbeit benutzt einen innovativen Paneldatensatz, im Gegensatz dazu wurde das ursprüngliche Modell anhand der Querschnittsdaten kalibriert. Das Schätzverfahren wurde weiterentwickelt, um die wiederholten Beobachtungen über ein normales Fehlerkomponentenmodell mit einer latenten Variablen im hierarchischen Bayes'schen Rahmenkonzept zu berücksichtigen. Zusätzlich wurde das *R* -Paket *nmm* entwickelt, um die Schätzung der gemeinsamen kontinuierlich-diskreten

Modelle einer breiteren Öffentlichkeit zugänglich zu machen. Bereits erwähnte Zeitindikatoren werden häufig bei der Kosten-Nutzen-Analyse von Verkehrsinfrastrukturprojekten verwendet. Aufgrund der unterschiedlichen Zeitzuweisungsmuster ist der ursprünglich geschätzte durchschnittliche Freizeitwert für Männer doppelt so hoch wie für Frauen. Um geschlechtsspezifische Vorurteile zu vermeiden, wurde vorgeschlagen, Hausarbeit als offizielle Arbeit zu behandeln, indem ihr der Marktlohnsatz zugewiesen wird. Nach dieser Einstellung verschwindet die Lücke fast vollständig.

Contents

1 Introduction

Human beings are constantly involved in the decision-making process. Even though artificial intelligence is becoming a bigger part of our life, we choose and will continue to choose not only what to do (go for a walk or visit a friend) but also the magnitude (long/short walk/visit). Thus, our decisions exist not only in the discrete choice but in the continuous space as well. Moreover, they are inter-and intra-connected with each other. The decision on sleep consists of other choices: to sleep or not to sleep; when to sleep; how long to sleep. If one sleeps longer, less time is left for eating, reading, or exercising; to save time one might choose to go to work by car. On contrary, if less time is assigned to sleeping, more time is available for other activities. However, because of the sleep deprivation, one might choose to use public transport this time and consume more coffee or food. All these choices can be expressed in statistical modeling as mathematical equations. They can be grouped into two independent systems of equations: continuous and discrete. Moreover, it is possible to merge them into the joint continuous-discrete choice model.

In the context of statistical modeling, continuous or discrete systems of equations can be estimated with various techniques. Continuous systems can be assessed as (non)linear multivariate models, i.e. seemingly unrelated regressions (SUR)(Zellner, 1962), the system of nonlinear regressions (SNR), (non)linear simultaneous equations model (SEM or NSEM). Whereas, the discrete choice models are estimated with logit, probit, and various derivatives of them. Due to their nonlinear functional form, these models too can be viewed as a part of the SNR family.

Linear multivariate models are widely used in research and have a strong literature background. Whereas, the main focus of this thesis is the estimation of a sub-sample of nonlinear multivariate models (NMM) - SNR, NSEM, logit. Moreover, it enables the unique joint estimation of SNR and logit in the `R` environment (R Core Team, 2020).

The evolution of this work went through these stages: (i) analysis and improvement upon the existing estimation method of the joint continuous-discrete model presented in Munizaga et al. (2008); (ii) implementation in a form of statistical package software `nmm` (Jokubauskaite et al., 2021) in `R` (R Core Team, 2020); (iii) application in estimating the advanced joint model presented in Jokubauskaitė et al. (2019) on the innovative Mobility-Activity-Expenditure data gathered in 2015 Austria (Aschauer et al., 2019, 2018); (iv) publication/submission of related papers (Schmid et al., 2019; Hössinger et al., 2019; Jokubauskaitė et al., 2019; Jokubauskaitė et al., 2021a,b).

The following Section (2) gives an overview of the theory related to the nonlinear models. The developed statistical software is presented in Section (3). Articles related to this thesis are introduced in Section (4). Section (5) offers an outlook of further research questions and Section (6) summarizes this thesis.

2 Nonlinear Multivariate Models

In this section, a theory related to the estimation of nonlinear models is introduced. It is based on the framework presented in MacKinnon and Davidson (1999) (Chapter 12). The specific method of the joint continuous-discrete model estimation is derived and improved upon the work of Munizaga et al. (2008). For theory related to multivariate linear models, please see Henningsen and Hamann (2007).

2.1 Systems of Nonlinear Regressions

A system of nonlinear regressions (SNR), which is as well called seemingly unrelated regressions (SUR), is defined by such equations:

$$y_{it} = x_{it}(\beta) + u_{it},\ \ i = 1, \ldots, g,\ \ t = 1, \ldots, n \tag{1}$$

These notations have the following meaning: y_{it} is the observation t of dependent variable i, $x_{it}(\beta)$ is observation t of the regression function i, β is a k-vector of parameters to be estimated and u_{it} is an error term with $E(u|X) = 0$. Here X stands for all explanatory variables that appear in all of the regression functions. It is assumed that error terms are serially uncorrelated, homoscedastic within each equation, and have contemporaneous covariance matrix Σ with elements σ_{ij}:

$$E(u_{it}, u_{jt}) = \sigma_{ij}, \forall t \tag{2}$$

$$E(u_{it}, u_{js}) = 0, \forall t \neq s \tag{3}$$

By stacking observations together, one can rewrite Equation (1) as:

$$y_i = x_i(\beta) + u_i,\ \ E\left(u_i u_j^{\top}\right) = \sigma_{ij} I_n,\ \ i, j = 1, \ldots, g \tag{4}$$

I_n is a $n \times n$ identity matrix. Under the normality assumption of errors, one can formulate the log-likelihood function as:

$$\begin{aligned} LL(\beta) = \log(L(\beta)) = &- \frac{gn}{2} \log 2\pi - \frac{n}{2} \log|\Sigma| - \\ &- \frac{1}{2} (y_{.} - x_{.}(\beta))^{\top} \left(\Sigma^{-1} \oplus I_n\right) (y_{.} - x_{.}(\beta)) \end{aligned} \tag{5}$$

In the equation above, $y_{.}$ denotes gn-vector of y_is stacked vertically, $x_{.}(\beta)$ denotes gn-vector of $x_i(\beta)$s stacked in the same way. The covariance matrix of error terms based on the maximum likelihood estimation is given by the formula:

$$\hat{\Sigma}_{ML} = \frac{1}{n} U^{\top} \left(\hat{\beta}_{ML} \right) U \left(\hat{\beta}_{ML} \right) \tag{6}$$

$U \left(\hat{\beta}_{ML} \right)$ is $n \times g$ matrix with columns equal to $\left(y_i - x_i \left(\hat{\beta}_{ML} \right) \right)$.

2.2 Nonlinear Simultaneous Equation Model

In the case of simultaneous equations, the model is formulated in the following way:

$$f_{it} \left(Y_t, \beta \right) = u_{it}, \ \ i = 1, \ldots, g, \ \ t = 1, \ldots, n \tag{7}$$

Here, f_{it} are nonlinear functions of the predetermined endogenous variables Y_t ($1 \times g$ vector) and parameters - β (k-vector). $f_i \left(Y, \beta \right)$ is n-vector with an element t equal to $f_{it} \left(Y_t, \beta \right)$ and Y is the $n \times g$ matrix with row t equal to Y_t. We can stack $f_i \left(Y, \beta \right)$ horizontally to get $h_t \left(Y_t, \beta \right)$, which is $1 \times g$ row vector containing the elements $f_{1t}, \ldots, f_{gt}$. Now Equation (7) with normally independently distributed (NID) errors can be rewritten as:

$$h_t \left(Y_t, \beta \right) = U_t, \ \ U_t \sim NID \left(0, \Sigma \right) \tag{8}$$

Model presented in Equation (8) has the following concentrated (with respect to Σ) log-likelihood functions:

$$-\frac{gn}{2} \left(\log 2\pi + 1 \right) + \sum_{t=1}^{n} log \left| det J_t \right| - \frac{n}{2} log \left| \frac{1}{n} \sum_{t=1}^{n} h_t^{\top} \left(Y_t, \beta \right) h_t \left(Y_t, \beta \right) \right| \tag{9}$$

Here, J_t is the Jacobian matrix and $J_t = \partial h_t \left(\beta \right) / \partial Y_t$ is different $\forall t$.

2.3 Multinomial Logit

Joint estimation of the continuous-discrete model makes up the core of this thesis. The continuous part is represented by a system of nonlinear equations that were discussed previously. This subsection focuses on the estimation of the discrete model, namely logit.

The individual utility is maximized by choosing the alternative q:

$$U_q = V_q + \epsilon_q \geq max_{m \neq q} \{U_m\} \tag{10}$$

$$V_q \geq max_{m \neq q} \{U_m\} - \epsilon_q = w_q \tag{11}$$

Utility U consists of the observable part V - indirect utility - and an error term ϵ. V is a function of parameter vector θ. w_q follows the logistic distribution under the assumption that ϵ_q terms are Gumbel distributed(Domencich et al., 1975). In this case, the distribution function is the following probability:

$$P_q(\theta) = F(V_q(\theta)) = \frac{exp(V_q(\theta))}{\sum_{j=1}^{Q} \alpha_j exp(V_j(\theta))} \tag{12}$$

Here Q is a number of the possible choices, α_j is a dummy variable equal to 1 if alternative j is available and 0 otherwise. If repeated measurements are allowed the corresponding log-likelihood function can be written as:

$$LL(\theta) = \sum_{i=1}^{J} \sum_{k \in A_{ik}} \sum_{q=1}^{Q} \delta_{ikq} \alpha_{ikq} \log\left(P_{ikq}(\theta)\right) \tag{13}$$

Here J is the number of individuals, A_{ik} is the index set of choices that individual i made. The latter one was introduced to allow multiple measurements per person. δ_{ikq} is equal to 1 if alternative q was selected by the person i in her/his k-th choice situation and 0 otherwise. α_{ikq} is equal to 1 if alternative q is available for k-th choice of person i and 0 otherwise.

2.4 Joint Continuous-Discrete Distribution

Models given by Equations (4) and (12) can be estimated separately as well as jointly if simultaneous data is available for both frameworks. The most prominent applications of the latter: (i) the joint time-use and mode choice model (Jara-Díaz and Guevara, 2003; Jara-Díaz et al., 2008; Munizaga et al., 2008; Jokubauskaitė et al., 2019); (ii) the joint vehicle type choice and miles of travel model (Spissu et al., 2009). Derakhshan et al. (2015) offer a comprehensive summary of joint models applied in the fields of energy and transportation.

To combine these two systems of different types, the approach proposed by Lee (1983) was used in Munizaga et al. (2008). Following this method, a normal quantile transformation is applied to the probabilities of logit (Equation 12). Let w_{iq} be an error term w_q from Equation (11) for individual i and D_{iq} be equal to one if this individual chose alternative q and zero otherwise. Thus, it holds:

$$D_{iq} = 1 \leftrightarrow V_{iq} \geq w_{iq} \tag{14}$$

$w_i q$ is transformed into a standard normal term by applying the inverse normal function - Φ^{-1}. Φ^{-1} and F_q are monotonic transformations, hence the above inequality can be written as:

$$D_{iq} = 1 \leftrightarrow y_q = \Phi^{-1}(F_q(V_q)) \geq \Phi^{-1}(F_q(w_q)) = w_q^* \sim \mathcal{N}(0,1) \tag{15}$$

Using this, the probability to choose mode q is:

$$D_{iq} = 1 \leftrightarrow P_q = P(y_q \geq w_q^*) = P(w_q^* \leq y_q) = F_{w_q^*}(y_q) = \Phi(y_q) \tag{16}$$

Thus, the probability to choose mode q ($P(U = q)$) is equal to $\Phi(y_q)$.

This transformation together with the definition of the mixed joint density allows deriving the joint continuous-discrete density.

By definition of the mixed joint density, if one variable (Y_C) follows a continuous distribution and another (Y_D) one follows a discrete one, then the joint density has the following form:

$$f_{Y_C,Y_D}(y_C, y_D) = f_{Y_C|Y_D}(y_C|y_D)P(Y_D = y_D) = P(Y_D = y_D|Y_C = y_c)f_{Y_c}(y_C) \quad (17)$$

This notation differs from the original one proposed in Lee (1983). It is based not on the censored regression but uses the definition of the mixed joint density directly. Under the normality assumption of Y_C both approaches lead to the same expressions but the mixed joint density approach is more straightforward.

The likelihood for the joint continuous-discrete model is produced following the steps described in the scheme below:

$$\underbrace{u = (u_1, u_2, \dots u_g)', \mathcal{U} \sim \mathcal{N}(0, \Sigma)}_{\text{continuous block}}$$

$$\underbrace{V_q \geq max_{m \neq q} \{U_m\} - \epsilon_q = w_q, W \sim Logistic}_{\text{discrete block}}$$

$$\Downarrow$$

Apply F($\cdot$) and $\Phi(\cdot)^{-1}$ to the inequality above:

$$\Downarrow$$

$$y_q = \underbrace{\Phi^{-1}(F(V_q)) \geq \Phi^{-1}(F(w_q)) = w_q^*}_{\text{Quantile transformation}}$$

$$W^* \text{ and } Y \sim \mathcal{N}(0, 1)$$

$$\Searrow \qquad \Swarrow$$

The mixed joint density function:

$$f_{\mathcal{U},U}(u, q) = P(U = q|\mathcal{U} = u) f_{\mathcal{U}}(u)$$

$$\Downarrow$$

$$f(u, w_q^* \leq y_q) = P(w_q^* \leq y_q|u)\phi(u) = \Phi(y_q|u)\phi(u)$$

$$\Downarrow$$

Get conditional moments using the "Conditional normal distribution" theorem:

$$\Downarrow$$

$$E[y_q|u] = \mu_{y_q} + \Sigma_{y_q u}\Sigma_{uu}^{-1}(u - \mu_u)$$
$$Cov[y_q|u] = \Sigma_{y_q y_q|u} = \Sigma_{y_q y_q} - \Sigma_{y_q u}\Sigma_{uu}^{-1}\Sigma_{y_q u}$$

More information on conditional moments can be found in Appendix (A.2). Finally, the log-likelihood for the joint continuous-discrete model can be expressed as:

$$LL(\theta) = \sum_{i}^{J} \sum_{k \in A_k} \sum_{q}^{Q} \delta_{ikq}\alpha_{ikq} f(u_i(\theta))\Phi(y_{ikq}(\theta)|u_i(\theta)) \tag{18}$$

Here θ is a vector of parameters from continuous and discrete equations. Appendix (A.1) and Jokubauskaitė et al. (2021b) give an explicit formulation of log-likelihood with three continuous equations.

In Jokubauskaitė et al. (2019), the advanced joint continuous-discrete model was estimated with Mobility-Activity-Expenditure-Diary data gathered in 2015 in

Austria. This data set includes information on continuous variables of time-use and expenditures as well as discrete travel mode choices. Repeated observations were gathered only for the latter. By factoring in this panel structure, the joint continuous-discrete model was advanced. This was done in the Bayesian framework by employing a normal error component model with a latent variable (Walker et al., 2007). In the joint continuous-discrete choice model (Jokubauskaitė et al., 2021b), the error term ϵ_q from Equation (11) is updated to have the following form for alternative q and individual i:

$$\epsilon_{iq} = f_q \zeta_i + \nu_q \tag{19}$$

where ζ_i is a $(n_i \times 1)$ vector of i.i.d. standard normal variables (individual characteristics), f_q are alternative-specific factor loadings, and ν_q is a vector of Gumbel distributed errors (Walker et al., 2007; Toledo et al., 2007). n_i is the number of repeated observations for individual i.

The estimation of the joint continuous-discrete model is implemented as a two-stage procedure. In the first step, coefficients θ from Equation (18) are estimated using the Maximum Likelihood (ML) framework. In the second step, parameter vector θ is fixed and log-likelihood (Equation 18) is maximized with respect to the distributional parameters - variances and correlations of the error terms. Then, the variance-covariance matrix of the parameters - θ and distributional - is estimated using one of the methods described in the next subsections.

2.5 Estimation of Covariance matrix

Covariance matrix $\hat{V}\left(\hat{\beta}_{ML}\right)$ can be estimated with multiple methods. In this work, four of them are presented: (i) inverse of Hessian, (ii) outer-product-of-gradient, (iii) robust, (iv) robust-clustered. These methods can be applied to log-likelihoods given by Expressions (5), (9), (13), or (18). Calzolari and Panattoni (1988) compared different approaches and did not find a superior one.

2.5.1 Inverse of Hessian:

$$V\left(\hat{\beta}\right) = \left[-H\left(\hat{\beta}\right)\right]^{-1} = -\left(\frac{\partial^2 LL\left(\hat{\beta}\right)}{\partial\hat{\beta}\partial\hat{\beta}'}\right)^{-1} \tag{20}$$

Here H is the Hessian matrix $(k \times k)$ of log-likelihood with respect to β and k is the number of parameters. This estimator is referred to as the empirical Hessian estimator.

2.5.2 Outer-Product-of-the-Gradient

Another option is to use the outer-product-of-the-gradient proposed in Berndt et al. (1974a):

$$V\left(\hat{\beta}\right) = \left(\sum_{t=1}^{T} g_t\left(\hat{\beta}\right)' g_t\left(\hat{\beta}\right)\right)^{-1} = \left(G\left(\hat{\beta}\right)' G\left(\hat{\beta}\right)\right)^{-1} \tag{21}$$

Here g_t is the gradient values $(1 \times k)$ of the log-likelihood function, G is the gradient matrix $(T \times k)$ and T is the number of observations.

2.5.3 Robust

The so-called sandwich estimator is used to get the robust standard errors:

$$V\left(\hat{\beta}\right) = H^{-1}\left(\hat{\beta}\right) G\left(\hat{\beta}\right)' G\left(\hat{\beta}\right) H^{-1}\left(\hat{\beta}\right) = B\left(\hat{\beta}\right) M\left(\hat{\beta}\right) B\left(\hat{\beta}\right) \tag{22}$$

Here $M(\hat{\beta}) = G(\hat{\beta})'G(\hat{\beta})$ is “meat” in the sandwich estimation and $B(\hat{\beta}) = H^{-1}(\hat{\beta})$ is the “bread”.

2.5.4 Robust-Clustered

If one is dealing with repeated observations, the robust-clustered covariance matrix can be calculated by adapting the sandwich estimator. The so-called "meat" ($M(\hat{\beta})$) layer is modified by first calculating the sum over the groups/clusters and then calculating the final sum/"meat"($M_c(\hat{\beta})$):

$$M_c(\hat{\beta}) = \sum_{i=1}^{n(c)} \sum_{c=1}^{C} g_{ic}(\hat{\beta})' g_{ic}(\hat{\beta}) \tag{23}$$

$$V(\hat{\beta}) = B(\hat{\beta}) M_c(\hat{\beta}) B(\hat{\beta}) \tag{24}$$

Here C is the number of clusters, $n(c)$ is the number of observations in cluster c, g_{ic} is $1 \times k$. Clusters can be seen as individuals with repeated observations. $n(c)$ is allowed to be unequal in different clusters, meaning that one cluster can have 4 observations and another 10.

3 Application as an R package - nmm

The joint estimation procedure presented in Section (2.4) was implemented as an `R` (R Core Team, 2020) package - `nmm` (Jokubauskaite et al., 2021) (Nonlinear Multivariate Models). Although the estimation of separate parts of this model was already possible in `R`, the joint estimation was missing. `nlsystemfit` function from the `systemfit` package (Henningsen and Hamann, 2007) can be used to estimate a System of Nonlinear Regressions (SNR) presented in Section (2.1). The multinomial logistic regression (logit) can be estimated with multiple `R` packages: `mlogit` (Croissant, 2019), `mnlogit` (Hasan et al., 2016), `MNP` (Imai and van Dyk, 2017), `nnet` (Venables and Ripley, 2002), etc. The joint estimation of SNR and logit is now made available with the `nmm` package. It has been used so far in several published works: Jokubauskaitė et al. (2019), Hössinger et al. (2019).

Both models - the SNR and logit - are treated as nonlinear systems of equations (NSE). Thus, the `nmm` package can be used not only for the estimation of the joint continuous-discrete model but also for the separate parts: either SNR or multinomial logit. Variance-covariance matrix can be calculated using different methods: normal, robust, robust-clustered (Section 2.5). Moreover compared to the `nlsystemfit` function, `nmm` offers a more user-friendly summary output for the nonlinear models. It also allows a flexible estimation of logit: individual-specific (non-linear) indirect utility functions with or without interaction terms. More detailed usage of the `nmm` package is available in Jokubauskaitė et al. (2021b).

3.1 nmm usage

Estimation of time-use and expenditure model as in Jara-Díaz and Guevara (2003) is used for the brief demonstration of the `nmm` functionality. The following system of nonlinear equations is estimated:

$$T_w = \frac{\left((\Phi+\theta_w)(\tau-T_c)+(1+\theta_w)\frac{E_c}{w}\right) \pm \sqrt{D}}{2(1+\Phi+\theta_w)}$$

$$T_{fi} = \frac{\theta_i}{1}(\tau - T_w - T_c),\ \text{i=\{1, 2\}}$$

$$E_{fj} = \frac{\phi_j}{\Phi}(wT_w - E_c),\ \text{j=\{1, 2, 3\}}$$

$$\text{Here } D = \left((\Phi+\theta_w)(\tau-T_c)+(1+\theta_w)\frac{E_c}{w}\right)^2 - 4(1+\Phi+\theta_w)\theta_w(\tau-T_c)\frac{E_c}{w}$$

Amount of work hours are given by T_w, T_{fi} represents the duration of leisure activity group$i$ and E_{fj} is freely spent money on expenditure group j. T_c and E_c are exogenously given time and money spent on committed activities/goods. τ is the total available time. $\{\theta_w, \Phi, \theta_1, \phi_1, \phi_2\}$ are parameters that are estimated, other parameters $\{\theta_2, \phi_3\}$ are calculated based on the binding expressions given in Jara-Díaz and Guevara (2003). Such a system of equations was used in the works of Hössinger et al. (2019), Jokubauskaitė et al. (2019) or Jokubauskaitė et al. (2021a) and originates from Jara-Díaz and Guevara (2003).

To estimate this model, data set `MAEDtimeExpenditure` from the `nmm` package is used. It was collected in 2015 in Austria. This data set is one of this kind, as it includes simultaneous information on time-use, expenditure, and travel mode choice. Week cycle of 737 individuals was observed. Variables related to activities have names starting with T and the ones representing expenditure groups - with E. Multi-faceted analyses of `MAEDtimeExpenditure` are provided in Aschauer et al. (2018), Aschauer et al. (2019), and Hössinger et al. (2019).

In the first step of the estimation procedure, equations and parameters of interest are defined:

```
library(nmm)
eq_c <- c(
    "Tw ~ ((((PH) + (tw)) * (ta - Tc) + Ec/w * (1 + (tw)) +
    sqrt((Ec/w *(1 + (tw)) + (ta - Tc) * ((PH) + (tw)))^2 -
    4 * Ec/w * (ta -Tc) * (tw) * (1 + (PH) + (tw))))/(2 *
    (1 + (PH) + (tw))))",
  "Tf1 ~ (th1) * (ta - (Tw) - Tc)",
  "Ef1 ~ (ph1)/(PH) * (w*(Tw) - Ec) ",
  "Ef2 ~ (ph2)/(PH) * (w*(Tw) - Ec)")
par_c <- c("tw", "PH", "th1", "ph1", "ph2")
```

Then T_w is substituted into the other equations:

```
eq_c[-1] <- gsub("Tw", gsub(".*~", "",eq_c[1]), eq_c[-1])
```

Estimation is performed with the main `nmm` function:

```
r_cont <- nmm(data = MAEDtimeExpenditure, eq_c = eq_c,
              par_c = par_c, eq_type = "cont")
```

Argument `eq_type` is set to “cont”, because continuous system of nonlinear equations is estimated. Method `summary` provides a nice looking summary of the estimated model:

```
summary(r_cont)
```

```
--------------------------------------------
Maximum Likelihood estimation
BFGS maximization, 23 iterations
Return code 0: successful convergence
Log-Likelihood: -8820.291
  free parameters
Estimates:
     Estimate Std. error t value Pr(> t)
PH   0.417160   0.027616  15.106  <2e-16 ***
ph1  0.262488   0.017178  15.280  <2e-16 ***
ph2  0.085480   0.005866  14.571  <2e-16 ***
th1  0.733412   0.004206 174.381  <2e-16 ***
tw  -0.617428   0.088474  -6.979  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
--------------------------------------------
```

Using these results, one can calculate the value of leisure (VoL) using this formula from Hössinger et al. (2019):

$$VoL = \frac{w\widehat{T_w} - E_c}{\widehat{\Phi}(\tau - \widehat{T_w} - T_c)}$$

This indicator shows "the valuation that the individual assigns to the liberated time" (Jokubauskaitė et al., 2019). Data used for the estimation and estimated parameters are combined into one data frame:

```
data4est <- cbind(attributes(r_cont)$data,
                  data.frame(t(r_cont$estimate)))
```

Next T_w expression is substituted into the VoL expression:

```
vol <- "(w * (Tw) -Ec)/(PH*(ta- (Tw)-Tc))"
vol <- gsub("Tw",gsub(".*~", "", eq_c[1]), vol)
```

Finally, VoL is calculated:

```
volest <- mean(eval(parse(text = vol), data4est))
volest
```

```
[1] 7.120091
```

This value shows that on average one hour of leisure is worth €7.12.

More detailed instructions on the usage of the `nmm` package can be found in Jokubauskaitė et al. (2021b) and an example of the Bayesian estimation is presented in Appendix (A.3).

4 Research articles

In 2015, an innovative Mobility-Activity-Expenditure-Diary (MAED, Aschauer et al., 2019) was developed in Austria. It resulted in a unique data-set was gathered (Aschauer et al., 2018), which provided information on different indicators: the distribution of time, expenditures, travel mode choice, duration and cost of trips, etc. It also inspired an advancement in the estimation procedure of the joint time-use-expenditure and travel mode choice model presented in Munizaga et al. (2008). The software developed as an `R` package called `nmm` (Jokubauskaite et al., 2021) and used in the estimation enabled to take into account the panel structure of the MAED data. `nmm` also allows specifying the individual-specific heterogeneous number of trips and transport mode-specific functional form. It also introduced the unique joint continuous-discrete model estimation into the `R` environment (Jokubauskaitė et al., 2021b).

As a result strand of publications appeared with this unique data set or/and the novel `nmm` package. Various value of time indicators were estimated in them: the value of leisure (VoL); the value of time assigned to work (VTAW); the value of time assigned to travel (VTAT); and the value of travel time savings (VTTS). These indicators can be used by the policymakers in the cost-benefit analysis of transport-related infrastructure projects.

The estimation procedure presented in this work (2) and in Jokubauskaitė et al. (2021b) started with the time-use model presented in Hössinger et al. (2019). This model was treated as a system of nonlinear regressions and estimated in the maximum likelihood framework. The main goal of that paper was to calculate the VoL, VTAW, and VTAT. The first two indicators were reported with standard errors, the last one without. The VTAT is calculated as a difference between the VoL and the VTTS. The values of the VTTS came from another model presented in Schmid et al. (2019), which followed a different estimation technique compared to Jokubauskaitė et al. (2021b). The joint continuous-discrete model with panel structure, which was introduced in Jokubauskaitė et al. (2019), allowed to get the VTAT estimates with standard errors. Additionally, it was noticed that the men's VoL was almost double of women - 10.63€/h and 5.86€/h, respectively. After a detailed investigation of the data, it was observed that these two groups mainly differ in the time distribution, namely, in time spent on the official work and housework. Women spent on average 8 hours more on household chores and about equally less on the official work. As domestic work does not generate income, the theoretical time-use model ignores the non-monetary value generated by this type of work. Jokubauskaitė et al. (2021a) proposed to assess domestic work based on the "third-person" principle (Reid, 1934) and to add it to the monetary value of the official work. After this adjustment, the gender-specific gap in the estimated VoL diminished - 9 €/h for both genders. What is more, Jokubauskaitė et al. (2021a) used a new approach for the valuation of domestic work. Chore-specific wage data was gathered from online platforms to better reflect the market wage rate of domestic work. Following this idea, Jokubauskaitė and Schneebaum (2021) estimated that in 2018 unpaid domestic work in Austria was additionally worth around 22% of the reported GDP.

Published Articles

- Simona Jokubauskaite, Reinhard Hössinger, Florian Aschauer, Regine Gerike, Sergio Jara-Díaz, Stefanie Peer, Basil Schmid, Kay WAxhausen and Friedrich Leisch (2019). Advanced continuous-discrete model for joint time-use expenditure and mode choice estimation. *Transportation Research Part B: Methodological*, 129:397-421, https://doi.org/10.1016/j.trb.2019.09.010
- Simona Jokubauskaitė and Alyssa Schneebaum (2021). Assessing the value of household work based on wages demanded on online platforms for substitutes. *Rev Econ Household*, https://doi.org/10.1007/s11150-021-09545-y (published online)
- Reihard Hössinger, Florian Aschauer, Sergio Jara-Díaz, Simona Jokubauskaitė, Basil Schmid, Stefanie Peer, Kay WAxhausen and Regine Gerike (2019). A joint time-assignment and expenditure-allocation model: value of leisure and value of time assigned to travel for specific population segments. *Transportation*, 47:1439-1475, https://doi.org/10.1007/s11116-019-10022-w
- Basil Schmid, Simona Jokubauskaite, Florian Aschauer, Stefanie Peer, Reinhard Hössinger, Regine Gerike, Sergio RJara-Diaz and Kay WAxhausen (2019). A pooled RP/SP mode, route and destination choice model to investigate mode and user-type effects in the value of travel time savings. *Transportation Research Part A: Policy and Practice*, 124:262-294, https://doi.org/10.1016/j.tra.2019.03.001

Submitted for Review

- Simona Jokubauskaitė, Reihard Hössinger, Sergio Jara-Díaz, Stefanie Peer, Alyssa Schneebaum, Basil Schmid, Florian Aschauer, Regine Gerike, Kay WAxhausen and Friedrich Leisch (2021). The role of unpaid domestic work in explaining the gender gap in the (monetary) value of leisure.
- Simona Jokubauskaitė, Friedrich Leisch and Reihard Hössinger (2021). The

nmm R package: Estimation of Nonlinear Multivariate Models.

Software

- Simona Jokubauskaite, Reinhard Hoessinger and Friedrich Leisch (2021). nmm: Nonlinear Multivariate Models. https://CRAN.R-project.org/package=nmm

5 Outlook

The `nmm` package estimates the following non-linear models - a system of nonlinear regressions (SNR), logit, and a joint continuous-discrete model - in the maximum likelihood framework. It uniquely allows to factor in the correlations between error terms of the SNR and logit.

Logistic regression (logit) is based on the independence of irrelevant alternatives assumption (IIA), which sometimes might be doubtful. This package could be further extended and include the estimation of probit, which relaxes the IIA assumption. This would require to use the copula decomposition.

Additionally, `nmm` could be extended to use more complicated parameter restrictions. Currently, it is possible to impose only equality constraints between the parameters by explicitly adjusting the equation system. After such extension, one could apply `nmm` for the estimation of the (non)linear demand systems (AIDS, QUAIDS).

What is more, `nmm` employs optimizers used by the `maxLik` package: "NR" (Newton-Raphson), "BFGS" (Broyden-Fletcher-Goldfarb-Shanno, Broyden, 1970; Fletcher, 1970; Goldfarb, 1970; Shanno, 1970), "BHHH" (Berndt-Hall-Hall-Hausman, Berndt et al., 1974b), "SANN" (Simulated ANNealing, Bélisle, 1992), "CG" (Conjugate Gradient, Hestenes et al., 1952), or "NM" (Nelder-Mead, Nelder and Mead, 1965). Most of them are local optimizers, except SANN, and thus optimization might get stuck on a flat surface of the likelihood function. This is partially solved in the `nmm` package by the usage of the evolutionary global optimization via the

Differential Evolution algorithm (Mullen et al., 2011) as an intermediate step in the optimization (control parameter `DEoptim_run_main`).

A different route could be taken by going fully away from the frequentist to the Bayesian inference. Some groundwork is already laid in the `nmm` package. It includes an option to form the log-likelihood functions by setting argument `bayesian_random = TRUE` and `estimate = FALSE`. The output from the `nmm` function, namely the attribute of the `nmm` object (`attributes(object)$functions$jfunc`) then could be used with the `RSGHB` package (Dumont et al., 2015), which uses the hierarchical Bayesian estimation framework. An example of such a procedure is provided in Appendix (A.3).

6 Summary

Statistical modeling approximates and simplifies the complex reality by giving it a form of a mathematical equation. Linear regression is a basic and the most commonly used technique. It mirrors the real-world processes via a linear relationship between covariates and predictors onto the real number line. Zellner (1962) defined the seemingly unrelated regression, which enabled the simultaneous modeling of multivariate processes. There exist numerous types of multivariate models that can be divided into two basic categories: seemingly unrelated non-linear regressions and non-linear simultaneous equation models. Both types include models with either continuous or discrete dependent variables. Disjoint estimation of these multivariate models is available with different `R` (R Core Team, 2020) packages: continuous with Henningsen and Hamann (2007) or Taha (2015); discrete with Croissant (2019); Hasan et al. (2016); Imai and van Dyk (2017); Venables and Ripley (2002) and others. `nmm` (Jokubauskaite et al., 2021) package introduced in this thesis enables the joint estimation of discrete-continuous models.

This cumulative work (i) summarized theory related to the estimation of non-linear multivariate models (NMM); (ii) focused on the estimation of such model in `R` environment (Jokubauskaitė et al., 2021b); (iii) resulted in an `R` package `nmm` (Non-linear Multivariate Models, Jokubauskaite et al., 2021); (iv) applied `nmm` to get the value of different time indicators in several published papers (Hössinger et al., 2019; Jokubauskaitė et al., 2019; Jokubauskaitė et al., 2021a). Using the maximum likelihood framework, the `nmm` package estimates these non-linear multivariate models: a system of non-linear regressions, multivariate logit, and the joint model of both of them. `nmm` is an important step in the evolution of the NMM estimation in the `R` environment and lays a foundation for future research in this field.

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A Appendix

A.1 Log-likelihood

Assume one has three continuous equations, whose error terms are denoted as η_i, and Q discrete ones. The individual log-likelihood has the following form:

$$\begin{aligned}
LL = & \sum_{q=1}^{Q} \alpha_q \delta_q ln \left(\phi(\eta_1) \phi \left(\frac{\eta_2 - \mu_{\eta_2|\eta_1}}{\sigma_{\eta_2|\eta_1}} \right) \phi \left(\frac{\eta_3 - \mu_{\eta_3|\eta_1\eta_2}}{\sigma_{\eta_3|\eta_1\eta_2}} \right) \right. \\
& \left. \Phi \left(\frac{y_q - \mu_{y_q|\eta_1\eta_2\eta_3}}{\sigma_{y_q|\eta_1\eta_2\eta_3}} \right) \right) = \\
& = ln \left(\phi(\eta_1) \right) + ln \left(\phi \left(\frac{\eta_2 - \mu_{\eta_2|\eta_1}}{\sigma_{\eta_2|\eta_1}} \right) \right) + ln \left(\phi \left(\frac{\eta_3 - \mu_{\eta_3|\eta_1\eta_2}}{\sigma_{\eta_3|\eta_1\eta_2}} \right) \right) + \\
& + \sum_{q=1}^{Q} \alpha_q \, \delta_q \; ln \left(\Phi \left(\frac{y_q - \mu_{y_q|\eta_1\eta_2\eta_3}}{\sigma_{y_q|\eta_1\eta_2\eta_3}} \right) \right)
\end{aligned} \tag{A.1}$$

Here Q is the number of alternatives, y_q equals $\Phi^{-1}(F(\theta))$ (the quantile transformed probability) and η_i is the error term from the i-th continuous equation which depends on the parameter vector θ. One can then rewrite the log-likelihood function as:

$$\begin{aligned}
LL(\theta) = ln \left(\phi(\eta_1(\theta)) \right) + \quad & ln \left(\phi \left(\frac{\eta_2(\theta) - \mu_{\eta_2|\eta_1}(\theta)}{\sigma_{\eta_2|\eta_1}} \right) \right) + \\
& + ln \left(\phi \left(\frac{\eta_3(\theta) - \mu_{\eta_3|\eta_1\eta_2}(\theta)}{\sigma_{\eta_3|\eta_1\eta_2}} \right) \right) + \\
& + \sum_{q} \alpha_q \, \delta_q \; ln \left(\Phi \left(\frac{\Phi^{-1}(F(\theta)) - \mu_{y_q|\eta_1\eta_2\eta_3}(\theta)}{\sigma_{y_q|\eta_1\eta_2\eta_3}} \right) \right)
\end{aligned} \tag{A.2}$$

The first three terms are easily differentiable. The problem appears only with the part:

$$ln\left(\Phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu_{y_q|\eta_1\eta_2\eta_3}(\theta)}{\sigma_{y_q|\eta_1\eta_2\eta_3}}\right)\right) = ln\left(\Phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu(\theta)}{\sigma}\right)\right) \tag{A.3}$$

Here $\mu(\theta) = \mu_{y_q|\eta_1\eta_2\eta_3}(\theta)$ and $\sigma = \sigma_{y_q|\eta_1\eta_2\eta_3}$.

A.1.1 The first derivative (gradient)

To derive the first derivative, we will use:

$$\frac{\partial}{\partial\theta} ln\left(\Phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu(\theta)}{\sigma}\right)\right) = \frac{1}{\sigma}\left(\frac{t2(\theta)\, t4(\theta)}{t1(\theta)\, t3(\theta)} - \frac{t2(\theta)\, t5(\theta)}{t1(\theta)}\right) \tag{A.4}$$

$$y = f(x) \tag{A.5}$$

$$f^{-1}(y) = x, \text{ due to } f^{-1}(f(x)) = x \tag{A.6}$$

$$\frac{\partial f^{-1}(y)}{\partial x} = \frac{\partial x}{\partial x} = 1 \tag{A.7}$$

If the argument of $f^{-1}$is variable y:

$$\frac{\partial f^{-1}(y)}{\partial y}\frac{\partial y}{\partial x} = 1 \tag{A.8}$$

$$\frac{\partial f^{-1}(y)}{\partial y} = \frac{1}{\frac{\partial y}{\partial x}} = \frac{1}{\frac{\partial f(f^{-1}(y))}{\partial x}} \tag{A.9}$$

If the argument of f^{-1} is a function $F(\theta)$:

$$\frac{\partial f^{-1}(F(\theta))}{\partial\theta} \overset{\text{chain rule}}{=} \frac{\partial f^{-1}(F(\theta))}{\partial F(\theta)}\frac{\partial F(\theta)}{\partial\theta} \overset{(A.9)}{=} \frac{1}{\frac{\partial f(f^{-1}(F(\theta)))}{\partial\theta}}\frac{\partial F(\theta)}{\partial\theta} \tag{A.10}$$

$$\frac{\partial f^{-1}(F(\theta))}{\partial F(\theta)} = \frac{1}{\frac{\partial f(f^{-1}(F(\theta)))}{\partial\theta}} \tag{A.11}$$

Thus:

$$\begin{aligned}
&\frac{\partial}{\partial\theta} ln\left(\Phi\left(\frac{\Phi^{-1}(F(\theta))-\mu(\theta)}{\sigma}\right)\right) = \qquad \text{(A.12)}\\
&= \underbrace{\frac{1}{\Phi\left(\frac{\Phi^{-1}(F(\theta))-\mu(\theta)}{\sigma}\right)}}_{t1}\overbrace{\phi\left(\frac{\Phi^{-1}(F(\theta))-\mu(\theta)}{\sigma}\right)}^{t2}\\
&\underbrace{\left(\underbrace{\frac{1}{\phi\left(\Phi^{-1}(F(\theta))\right)}}_{t3}\overbrace{\frac{\partial F(\theta)}{\partial\theta}}^{t4}\frac{1}{\sigma} - \overbrace{\frac{\partial\mu(\theta)}{\partial\theta}}^{t5}\frac{1}{\sigma}\right)}_{\Delta} =\\
&= \frac{1}{t1(\theta)}\,t2(\theta)\underbrace{\left(\frac{1}{t3(\theta)}t4(\theta)\,\frac{1}{\sigma} - t5(\theta)\,\frac{1}{\sigma}\right)}_{\Delta} = \frac{1}{\sigma}\left(\frac{t2(\theta)\,t4(\theta)}{t1(\theta)\,t3(\theta)} - \frac{t2(\theta)\,t5(\theta)}{t1(\theta)}\right)
\end{aligned}$$

A.1.2 The second derivative (Hessian)

$$\begin{aligned}
&\frac{\partial^2}{\partial^2\theta} ln\left(\Phi\left(\frac{\Phi^{-1}(F(\theta))-\mu(\theta)}{\sigma}\right)\right) = \qquad \text{(A.13)}\\
&= \frac{1}{\sigma}\left(\underbrace{\frac{\partial}{\partial\theta}\left(\frac{t2(\theta)\,t4(\theta)}{t1(\theta)\,t3(\theta)}\right)}_{\text{T1}} - \underbrace{\frac{\partial}{\partial\theta}\left(\frac{t2(\theta)\,t5(\theta)}{t1(\theta)}\right)}_{T2}\right)
\end{aligned}$$

To get the expression of the second derivative we use:

$$(abc)' = a'bc + ab'c + abc' \tag{A.14}$$

$$\frac{ab}{cd} = \frac{(ab)'(cd) - (cd)'(ab)}{(cd)^2} = \frac{a'bcd + ab'cd - abc'd - abcd'}{(cd)^2} \tag{A.15}$$

$$\frac{ag}{c} = \frac{(ag)'cd - c'(ag)}{c^2} = \frac{a'gc + ag'c - agc'}{c^2} \tag{A.16}$$

A.1.2.1 T1

$$T1 = \frac{\partial}{\partial\theta}\left(\frac{t2(\theta)\,t4(\theta)}{t1(\theta)\,t3(\theta)}\right) = \frac{\Xi}{(t1(\theta)\,t3(\theta))^2} \tag{A.17}$$

Here:

$$a = t2(\theta), b = t4(\theta), c = t1(\theta), d = t3(\theta) \tag{A.18}$$

$$\begin{aligned}\Xi =& t2(\theta)'\,t4(\theta)\,t1(\theta)\,t3(\theta) + t2(\theta)\,t4(\theta)'\,t1(\theta)\,t3(\theta) - \\ &- t2(\theta)\,t4(\theta)\,t1(\theta)'\,t3(\theta) - t2(\theta)\,t4(\theta)\,t1(\theta)\,t3(\theta)'\end{aligned} \tag{A.19}$$

$$\begin{aligned}a' =& \frac{\partial t2(\theta)}{\partial\theta} = & \phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu(\theta)}{\sigma}\right)' = \phi(arg)' = \\ && \frac{\partial\phi(arg)}{\partial arg}\frac{\partial arg}{\partial\theta} = \frac{\partial\phi(arg)}{\partial arg}\Delta\end{aligned} \tag{A.20}$$

$$b' = \frac{\partial t4(\theta)}{\partial\theta} = \frac{\partial^2 F(\theta)}{\partial^2\theta} \tag{A.21}$$

$$c' = \frac{\partial t1(\theta)}{\partial \theta} = \Phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu(\theta)}{\sigma}\right) = t2(\theta)\Delta \tag{A.22}$$

$$d' = \frac{\partial t3(\theta)}{\partial \theta} = \phi\left(\Phi^{-1}(F(\theta))\right)' = \phi(arg)' = \frac{\partial \phi(arg)}{\partial arg}\frac{\partial arg}{\partial \theta} = \frac{\partial \phi(arg)}{\partial arg}\frac{t4(\theta)}{t3(\theta)} \tag{A.23}$$

A.1.2.2 T2

$$T2 = \frac{\partial}{\partial \theta}\left(\frac{t2(\theta)\, t5(\theta)}{t1(\theta)}\right) = \frac{t2(\theta)'\, t5(\theta)\, t1(\theta) \ + t2(\theta)\, t5(\theta)'\, t1(\theta) \ - t2(\theta)\, t5(\theta)\, t1(\theta)'\,'}{(t1(\theta)\,)^2} \tag{A.24}$$

Here:

$$a = t2(\theta), c = t1(\theta), g = t5(\theta) \tag{A.25}$$

$$a' = \frac{\partial t2(\theta)}{\partial \theta} = \phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu(\theta)}{\sigma}\right)' = \phi(arg)' = \frac{\partial \phi(arg)}{\partial arg}\frac{\partial arg}{\partial \theta} = \frac{\partial \phi(arg)}{\partial arg}\Delta \tag{A.26}$$

$$g' = \frac{\partial t5(\theta)}{\partial \theta} = \frac{\partial^2 \mu(\theta)}{\partial^2 \theta} \tag{A.27}$$

$$c' = \frac{\partial t1(\theta)}{\partial \theta} = \Phi\left(\frac{\Phi^{-1}(F(\theta)) - \mu(\theta)}{\sigma}\right) = t2(\theta)\Delta \tag{A.28}$$

A.2 Conditioning in the Multivariate Normal Distribution

Define $Z = (z_1, z_2, \ldots, z_k)$ as a vector of random variables that are jointly normally distributed:

$$f(Z) = (2\pi)^{k/2} |\Sigma|^{-1/2} exp \left[-\frac{1}{2}(Z-\mu)'\Sigma^{-1}(Z-\mu) \right] \tag{A.29}$$

μ is the vector of means and Σ is the contemporaneous covariance matrix. The joint normal distribution, can be partitioned as:

$$f(Z) = f(z_1) f(z_2|z_1) f(z_3|z_1, z_2) \ldots f(z_k|z_1, \ldots, z_{k-1}) \tag{A.30}$$

The theorem of conditional normal distribution helps to find the conditional normal distribution $f(z_i|z_1, \ldots, z_{i-1})$. Let $X = (X_1, \ldots, X_n)'$ and $Y = (Y_1, \ldots, Y_m)'$ be two multivariate normal variables of size n and m, respectively. The joint variable - $Z = (X_1, \ldots, X_n, Y_1, \ldots, Y_m)'$ is $(n+m)$ dimensional normal variable. Denote the expectations as follows:

$$E[X] = \mu_X, E[Y] = \mu_Y \tag{A.31}$$

and partition the covariance matrix for Z (with $\Sigma_{XY} = \Sigma'_{YX}$),

$$\Sigma = Cov\left[\begin{pmatrix} X \\ Y \end{pmatrix};\begin{pmatrix} X \\ Y \end{pmatrix}\right] = \begin{bmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_{YY} \end{bmatrix} \tag{A.32}$$

The conditional density of X given that $(Y = y)$ is:

$$f_{X|Y}(x,y) = \frac{f_{YX}(y,x)}{f_Y(y)}, \text{ thus } f_{YX}(y,x) = f_{X|Y}(x,y)\, f_Y(y) \tag{A.33}$$

Theorem. (Conditional normal distribution) The conditional normal distribution of X, given that $(Y = y)$, is n-dimensional normal with expectation and covariance matrix:

$$E[X|Y = y] = \mu_{X|Y} = \mu_X + \Sigma_{XY}\Sigma_{YY}^{-1}(Y - \mu_Y) \tag{A.34}$$

$$Cov\,(X|Y = y) = \Sigma_{XX|Y} = \Sigma_{XX} - \Sigma_{XY}\Sigma_{YY}^{-1}\Sigma_{XY} \tag{A.35}$$

Thus, $X|Y \sim \mathcal{N}\left(\tilde{\mu}, \tilde{\Sigma}\right)$, where $\tilde{\mu} = \mu_{X|Y}$ and $\tilde{\Sigma} = \Sigma_{XX|Y}$.

A.3 Bayesian estimation

1. Define controls for `doHB` function:

```
library(RSGHB)
control <- list(
  modelname =  "example",
  gVarNamesNormal = par_c,
  gDIST = rep(1, length(par_c)),
  svN = r_cont$estimate,
```

```
  gNCREP = 100,
  gNEREP = 200,
  gNSKIP = 1,
  gINFOSKIP = 100,
  verbose = FALSE,
  gSeed = 1987
)
```

2. Use `nmm` function to get the log-likelihood function:

```
bay_f <- nmm(data = MAEDtimeExpenditure, eq_c = eq_c, par_c = par_c,
              eq_type = "cont", estimate = FALSE,
             bayesian_random = TRUE)
ll <- attributes(bay_f)$functions$jfunc
data <- attributes(bay_f)$data
data$ID <- paste0(data$PeID)
data <- data[order(data$ID),]
```

Equations and parameters come from Subsection (3.1).

3. Define log-likelihood function, which will be used with `doHB` function from `RSGHB` package:

```
environment(ll) <- environment()
likelihood <- function(fc, b)
{
  b <- apply(b, 2, as.numeric)
  res <- ll(b) %>% rowSums
  if(any(res%in%c(NaN, Inf, -Inf))){
    replace(res, is.nan(res)|res%in%c(Inf, -Inf), NA)
  } else {
    exp(res)
  }
```

```
}
```

4. Check if function gives the same output:

```
stvr <- matrix(r_cont$estimate, ncol=length(r_cont$estimate),
               nrow=nrow(data), byrow=TRUE)
ll(stvr) %>% sum
```

```
[1] -8820.291
```

```
likelihood(b=stvr) %>% log %>%  sum
```

```
[1] -8820.291
```

```
r_cont$maximum
```

```
[1] -8820.291
```

5. Estimate:

```
model <- doHB(likelihood, data, control)
model$ll0
model$llf
```

This estimation could be used for the Bayesian estimation of the continuous model.

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Advanced continuous-discrete model for joint time-use expenditure and mode choice estimation

Simona Jokubauskaitė[a,*], Reinhard Hössinger[b], Florian Aschauer[b], Regine Gerike[c], Sergio Jara-Díaz[d], Stefanie Peer[e], Basil Schmid[f], Kay W. Axhausen[f], Friedrich Leisch[a]

[a] BOKU, Institute of Statistics, Peter-Jordan-Strasse 82, Vienna 1190, Austria
[b] BOKU, Institute for Transport Studies, Peter-Jordan-Strasse 82, Vienna 1190, Austria
[c] TU Dresden, Institute of Transport Planning and Road Traffic, Dresden 01069, Germany
[d] Universidad de Chile, Department of Civil Engineering and Instituto Sistemas Complejos de Ingeniería (ISCI), Casilla 228-3, Santiago, Chile
[e] WU, Institute for Multi-Level Governance and Development, Welthandelsplatz 1/D4, Vienna 1020, Austria
[f] ETH Zurich, Institute for Transport Planning and Systems (IVT), Stefano-Franscini-Platz 5, Zurich 8093, Switzerland

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ABSTRACT

This paper presents the joint time-use, expenditure and mode choice model, based on the theoretical framework of Jara-Díaz and Guevara (2003), for the first time estimated in panel setting while using surveyed expenditure data. This extended estimation takes into account multiple trips per individual, as well as mode availability. The model was estimated using the novel dataset gathered in Austria in 2015. It includes individual-specific information on time-use, expenditures and mode choice. As a result, we calculate the value of leisure (VoL), travel time savings (VTTS) and time assigned to travel (VTAT), that are relevant inputs to appraisals of transport policies. We also show that, at least for the Austrian working population, the omission of expenditures in the model might result in a significant overestimation of the value of leisure (16.83%); the VoL (9.29€/h) was estimated to be considerably lower than the wage rate (12.14€/h) and the VTTS varies strongly between the modes (9.98€/h for car, 3.91€/h for public transport, 9.25€/h for bike and 17.53€/h for walk). The joint estimation framework produced positive estimates of VTAT (5.38€/h) only for public transport, reflecting the favorable public transport conditions in Austria.

1. Introduction

The integration of travel decisions into the framework of time-use and activity scheduling has received increasingly more attention in recent years (for a detailed summary of different approaches, see e.g. Bhat, 1998; Bradley and Vovsha, 2005; Bhat et al., 2013; Jara-Díaz and Rosales-Salas, 2017). A prominent strand of research in this context was established by Jara-Díaz and Guevara (2003) and expanded by Jara-Díaz et al. (2008). They highlighted that a person who makes a travel

* Corresponding author.
E-mail address: simona.jokubauskaite@boku.ac.at (S. Jokubauskaitė).

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decision does not only maximize her/his utility in a particular choice situation, but also in the surrounding time-expenditure space. They developed a time-use framework model, which allows to estimate different aspects of time-use in monetary terms. A key output is the value of leisure (VoL). It represents the marginal utility of all activities with assigned time exceeding the necessary minimum. Following DeSerpa (1973), the VoL permits a deeper examination of the value of travel time savings (VTTS) obtained from travel choice models. The VTTS equals the VoL minus the value of time assigned to travel (VTAT). It summarizes the value of the liberated time (opportunity cost of travel), while the VTAT represents the direct utility (or disutility) derived from the time spent in the travel activity. The VTAT differs between modes and specific conditions of travel, such as comfort, reliability, crowding or the possibility to use in-vehicle time productively.

The VTAT is important from a transport planning perspective. For a public transport operator, it enables a comparative evaluation of investments in better travel conditions (supported by the VTAT) or in higher speed (justified by the VoL). Furthermore, the VTAT of car travel will presumably receive increasing attention in the context of autonomous driving: the release from the driving task enables secondary activities during the trip. As a result, time spent in a car will be perceived as being more useful (the VTAT will increase), and car use should become less sensitive to longer travel time, e.g. due to congestion.

So far some attempts have been made to estimate the model of Jara-Díaz et al. (2008), but the number of studies was limited by the large amount of required data. This model uses information about the patterns of time-use, expenditure allocation, and travel decisions. All of which have to be tracked over a whole work-leisure cycle (Jara-Díaz and Rosales-Salas, 2015; 2017). Appropriate datasets that cover such broad information were not available for a long time. Therefore, previous studies have estimated only incomplete models, mostly without travel decisions (Jara-Díaz et al., 2008; 2016), or considering only one trip (Munizaga et al., 2008). In order to overcome these limitations, Aschauer et al. (2019) developed a novel survey procedure, the so-called Mobility-Activity-Expenditure Diary (MAED). In 2015 it was applied for the first time while collecting the data of interest from employees in Austria.

Using the MAED data, Hössinger et al. (2019) provided the first results based on the complete modelling framework, including time-use and expenditure equations. The results include estimates for the VoL, VTTS and for the first time a mode-specific VTAT. A serious limitation, however, is that the results from the discrete choice model used in Hössinger et. al. (2019) come from the independent estimation done by Schmid et al. (2019). It is worth mentioning that both studies used the same dataset.[1] A consequence of the separate estimation is that possible correlations between the error terms of continuous and discrete decisions were not considered. Also, no confidence intervals were reported for the VTAT, as it was computed from the estimates of separate models.

This calls for a joint estimation procedure for both the continuous and discrete parts in order to obtain more efficient state-of-the-art estimates for all parameters and values of time. This is not possible in the multiple discrete-continuous extreme value model (MDCEV) proposed in Bhat (2005; 2008). The MDCEV can only be applied to decisions regarding activities that generate intrinsic positive utility. Travel is well known to be a derived activity which generates negative utility and which people would thus prefer to avoid. The time and expenses assigned to travel could enter the MDCEV only as an 'outside good', which is externally given but not estimated endogenously. We show the gains from the additional information and the joint estimation by comparing models with and without inter-block correlations (between time-use and travel mode decisions), and models with and without expenditure equation. The starting point for this study is the work by Munizaga et al. (2008), who presented a discrete-continuous model with explicit consideration of correlations between both types of decisions. The model was calibrated using a Chilean sample of long-distance commuters to downtown Santiago who completed a three days' activity diary. The dataset includes only one mode choice for the commuting trip and no expenditures. The resulting model is therefore incomplete in the sense that it includes no expenditure equation and a single morning trip to work, which is a very limited representation of the person's general travel behavior.

The objective of this paper is to improve over both Munizaga et al. (2008) and Hössinger et al. (2019) with three innovations. First, based on the aforementioned MAED dataset, we provide a joint estimation of the complete model framework – the time-use model and the travel choice model. It includes time-use equations, expenditure equations (for the first time using information from the same individuals) and all mode choices made over the whole observed period.[2] Second, the employed modelling framework allows the calculation of the value of leisure (VoL) along with different values of travel time (VTTS and VTAT). Third, we develop an advanced estimation procedure, which is able to use the rich information of the MAED dataset in a panel setting. The unobserved individual-specific characteristics that might affect the choices are modelled with latent factors. The joint estimation framework can also accommodate for the large and varying number of mode choices (MAED survey participants reported 23 trips on average during the survey week, each of which establishes a separate mode choice). Also, it is able to automatically derive the equations of the conditional moments (mean and standard

[1] The mode specific VTTS was estimated in a parallel effort by Schmid et al. (2019) from a discrete choice model, which combines different data types (RP, SP) and experiment types (mode, route, and shopping destination choice).

[2] Habib (2013) refers to such models with separate functions for discrete and continuous choices, which require that the correlation between both types of decisions needs to be modeled explicitly, as 'loosely coupled' – as opposed to 'tightly coupled' models, which use common attributes and parameters to estimate pairs of discrete and continuous choices (as a result, the juncture between both types is implicitly addressed and no extra measure is necessary to address the correlation). A prominent example of the latter is the multiple discrete-continuous extreme value model (MDCEV, see Bhat, 2005; Castro et al., 2012), which estimates the discrete choice, if a non-zero amount of time is assigned to a particular activity, and (if so), the continuous choice of the amount of time assigned to that activity.

deviation) of the normal distribution for a large number of variables. The complexity of these equations increases disproportionately to the number of variables, thus doing it manually might become a huge burden. The estimation solution was developed using the statistical computing language R (R Core Team, 2013).[3]

This paper is organized as follows. In Section 2, the theoretical joint model is introduced. The MAED data-set is discussed in Section 3. Section 4 contains the estimation results of the four models (with/without expenditure modelling with/whithout panel structure), as well as an *a-priori* segmentation analysis according to socioeconomic characteristics. Section 5 reviews the central findings of this study and discusses future research.

2. Modelling framework

DeSerpa (1971) proposed a sophisticated theoretical model, which treated utility as a function of commodities and time, and considered budget, total time, and minimal required time constraints. This model laid the foundation for the microeconomic model developed in Jara-Díaz and Guevara (2003). The authors combined travel mode choice and time allocation systems, and showed that "estimating both types of models from the same population makes it possible to obtain all components of the subjective value of travel time savings empirically" (pp. 29). Although Jara-Díaz et al. (2008) generalized the theoretical framework and presented a time-use-expenditure model, expenditures directly obtained from the same individuals were not used until Hössinger et al. (2019).[4] In our paper we further refine the modelling structure while using the methodology proposed by Jara-Díaz et al. (2008) and Munizaga et al. (2008). This approach takes into account not only the intra-continuous-block, but also the inter-block correlations (between time-use and travel mode decisions). We extend it with panel structure and the incorporation of the expenditure equation proposed in Jara-Díaz et al. (2008) and used in Jara-Díaz and Astroza (2013), as well as in Hössinger et al. (2019).

2.1. Time-use decision

2.1.1. Model formulation

Following the framework developed in Jara-Díaz and Guevara (2003) and Jara-Díaz et al. (2008), the agent's utility is assumed to have a Cobb-Douglas form:

$$U = T_w^{\theta_w} \prod_{i=1}^{n} T_i^{\theta_i} \prod_{j=1}^{m} E_j^{\phi_j} \tag{1}$$

In Eq. (1) utility U is a function of T_w - the amount of time assigned to work, T_i - the time assigned to activity i, and E_j - the expenditure assigned to good j. The exponents θ_w, θ_i, ϕ_j are the baseline utilities of time assigned to work, activities, and expenditures respectively. They also represent the elasticity of utility with respect to a corresponding input. The utility maximization problem can be expressed as:

$$\arg\max_{\theta_w,\theta^A,\phi^G} U = \arg\max_{\theta_w,\theta^A,\phi^G} ln(U) = \arg\max_{\theta_w,\theta^A,\phi^G} \left(\theta_w ln(T_w) + \sum_{i=1}^{n} \theta_i ln(T_i) + \sum_{j=1}^{m} \phi_j ln(E_j) \right) \tag{2}$$

subject to:

$$\tau - T_w - \sum_{i=1}^{n} T_i = 0 \ (\mu) \qquad \text{(time constraint)} \tag{3}$$

$$wT_w + I - \sum_{j=1}^{m} E_j \geq 0 \ (\lambda) \qquad \text{(budget constraint)} \tag{4}$$

$$T_i - T_i^{Min} \geq 0 \ (\kappa_i) \qquad \text{(technical constraints on activities)} \tag{5}$$

$$E_j - E_j^{Min} \geq 0 \ (\eta_j) \qquad \text{(technical constraints on goods)} \tag{6}$$

Here A and G are sets of activities and expenditures. θ^A and ϕ^G are vectors of time and expenditure exponents. Goods and activities are divided into two groups, freely chosen and committed. The latter ones restrict their freely chosen counterparts. Constraints (3)–(4) also include w - the wage rate, I - income not related to work, τ - total available time (in our study it will be 168 h). One can solve the presented maximization problem by applying the Lagrange method. The Lagrange multipliers are given on the right side of each constraint (μ, λ, κ_i, η_j). They show the marginal utility/cost of relaxing/strengthening the constraints. The technical constraints (Eqs. (5) and (6)) on those committed activities and goods

[3] A long-term objective is to provide the estimation procedure established for this paper as an R package for the estimation of discrete-continuous equation systems, because no package is available so far for this purpose.

[4] Expenditure equations were indeed used by Jara-Díaz and Astroza (2013) imputing expenses taken from other complementary sources.

that are necessary for personal and household maintenance (travel, rental cost etc.) are not explicitly estimated. They are inferred from the observations and introduced in the time and budget constraints as T_c and E_c (Hössinger et al., 2019). For these activities/goods consumers are left with no other choice but to stick to the technical minimum (T_i^{Min}/E_j^{Min}). The analytic solution of the constrained maximization problem defined by Eqs. (2)–(4) yields the following expressions of optimal amount allocated to labor, freely chosen activities, and expenditure groups (for more details, see Hössinger et al., 2019):

$$T_w^* = \frac{\left((\Phi+\theta_w)(\tau-T_c)+(\Theta+\theta_w)\frac{E_c}{w}\right)+\sqrt{D}}{2(\Theta+\Phi+\theta_w)} \tag{7}$$

$$\text{here } D = \left((\Phi+\theta_w)(\tau-T_c)+(\Theta+\theta_w)\frac{E_c}{w}\right)^2 - 4(\Theta+\Phi+\theta_w)\theta_w(\tau-T_c)\frac{E_c}{w}$$

$$T_i^* = \frac{\theta_i}{\Theta}(\tau - T_w^* - T_c) \tag{8}$$

$$E_j^* = \frac{\phi_j}{\Phi}(wT_w^* - E_c) \tag{9}$$

where $\Theta = \sum_{i\in A_f}\theta_i$, $\Phi = \sum_{j\in G_f}\phi_j$ with A_f and G_f being the index sets of freely chosen activities and goods. A_f^c and G_f^c are sets of committed activities and goods. $T_c = \sum_{i\in A_f^c} T_i^{min}$ and $E_c = \sum_{j\in G_f^c} E_j^{min}$ correspond to total committed time and expenditures, respectively.

2.1.2. Likelihood formulation

Under the assumption of normality, one can rewrite the system of Eqs. (7)–(9) as:

$$Y_i = g_i(\beta) + \eta_i,\ i \in \{1,\dots,3\} \tag{10}$$

where g_i is a function of parameter vector β and $\eta_i \sim N(\mu_i, \sigma_i)$ is error component. The estimation procedure takes into account the possible correlations between equations. Later, this dependency is referred to as intra-block correlation. The joint density can be partitioned as:

$$f(\eta) = f(\eta_1)f(\eta_2|\eta_1)f(\eta_3|\eta_1\eta_2) \tag{11}$$

The log-likelihood function for sample of size J is:

$$LL(\eta) = \sum_{i=1}^{J} ln(f(\eta_1)f(\eta_2|\eta_1)f(\eta_3|\eta_1\eta_2)) \tag{12}$$

Under the normality assumption of $\{\eta_1, \eta_2, \eta_3\}$, the conditional distributions as well as the conditional moments ($\mu_{\eta_2|\eta_1}$, $\mu_{\eta_3|\eta_1\eta_2}$, $\Sigma_{\eta_2|\eta_1}$, $\Sigma_{\eta_3|\eta_1\eta_2}$) can be found by applying the “Conditional Normal Distribution” Theorem.

2.1.3. Indicators

The maximum likelihood (ML) estimation for the given log-likelihood function (Eq. (12)) yields estimates of the parameters from Eqs. (7)–(9). Then using these values and the first order conditions from Oort (1969) or Jara-Díaz and Guevara (2003) the VoL and VTAW can be calculated as follows:

$$VoL = \frac{\partial U \backslash \partial T_i}{\partial U \backslash \partial E_j} = \frac{\mu}{\lambda} = \frac{\Theta(wT_w - E_c)}{\Phi(\tau - T_w - T_c)} \tag{13}$$

$$VTAW = VoL - w \tag{14}$$

where T_w is the fitted value of work time.

2.2. Discrete choice model

2.2.1. Model formulation

In the mode choice dimension, an individual again maximizes her/his personal utility and chooses transportation mode q if:

$$U_q = V_q + \epsilon_q \ge max_{m\neq q}\{U_m\} \tag{15}$$

$$V_q \ge max_{m\neq q}\{U_m\} - \epsilon_q = \omega_q \tag{16}$$

Utility U consists of the observable part V (the indirect utility) and an error term ϵ. The indirect utility V_q is assumed to be a function of duration ($time_q$), price ($cost_q$) and other mode specific variables. Following Munizaga et al. (2008) we assume that ϵ are Gumbel distributed and thus the new error term (ω) distributes logistically (Domencich et al., 1975). As not all transportation modes might be available for all observations, dummy variable α_{ikq} is introduced to control for this. The probability that the person i chooses mode q for her/his k-th trip is:

$$P_{ikq} = F(V_{ikq}) = \frac{exp(V_{ikq})}{\sum_j^Q \alpha_{ikj} exp(V_{ikj})} \tag{17}$$

where Q is the number of alternatives, and dummy variable α_{ikq} is equal to zero, if alternative q is not available for the trip k, and one otherwise.

2.2.2. Likelihood formulation

Under the assumption of Gumbel distributed errors, the log-likelihood is defined as follows:

$$LL(\theta) = \sum_{i=1}^{J}\sum_{k=1}^{n_i}\sum_{q=1}^{Q} \delta_{iq}\alpha_{ikq} \ln P_{ikq} \tag{18}$$

where J is the number of people, n_i is the total number of trips that person i has conducted.

2.2.3. Indicators

Using the results from Bates (1987) and Jara-Díaz and Guevara (2003), one can calculate the VTTS and the VTAT:

$$VTTS_q = \frac{\partial V_q \backslash \partial \mathrm{time}_q}{\partial V_q \backslash \partial \mathrm{cost}_q} \tag{19}$$

$$VTAT_q = VoL - VTTS_q \tag{20}$$

2.3. Joint estimation

Conceptually, the value of travel time savings (VTTS) estimated from travel choice models represents the willingness-to-pay to diminish travel time by one unit. As originally shown by DeSerpa (1971), the VTTS has two components: the opportunity cost regarding other activities (leisure or work) and the value of a reduction of the travel activity by itself. The first component is the value of leisure (VoL); the second, called the value of time assigned to travel (VTAT), depends on travel conditions. The analytical formula is given in Eq. (20). Here $VTTS_q$ is the (mode-specific) value of travel time saving, VoL the (individual-specific) value of leisure, and $VTAT_q$ the value of time assigned to travel, driven by mode-specific characteristics, such as comfort, and how productively in-vehicle time can be used for secondary activities (for a general derivation, see Jara-Díaz (2007), Chapter 2). The equation shows that unless one has an estimate of both, i.e. VoL and $VTTS_q$, the $VTAT_q$ simply cannot be estimated. This is exactly the reason why a joint model of time-use and mode choice is needed.

The joint estimation of the full model framework with all types of decisions (time-use, expenditures, and mode choice for all weekly trips) is the key innovation presented in this paper. The estimation advancements were partly forced, partly matured by the usage of the rich MAED dataset and the necessity to transform the available information into the continuous-discrete model variables. We advance the relevant literature along the following lines. Firstly, the expenditure (Eq. (9)) was included into the modelling framework, secondly the assumption of one trip per individual was given up. Thirdly, the derivation of the conditional moments was automated with a computer algebra system (Maxima) and R. This flexible procedure simplifies the inclusion of more than three equations into the continuous block, as well as the modelling of a high and variable number of discrete choices, and the usage of extended utility functions with interaction terms.

The previously defined systems of equations (Eq. (7)–(9)) and indirect utilities from Eq. (17) remain the same in this joint model; only the estimation procedure changes. As Munizaga et al. (2008) pointed out, the error terms from both system blocks, Section 2.1 and 2.2, may be correlated due to common parameters/variables or hidden relationships between the variables (e.g. the duration of trip influences how much free time is left). Because of this, it is desirable to jointly estimate both systems of equations and account for possible inter-block correlations. What is more, mode choice utility is a conditional indirect utility function that can be derived from a activity-consumption consumer behavior model (Jara-Díaz and Guevara, 2003; Jara-Díaz, 2007). So direct utility and the so-called modal utility have to be compatible, but they belong to different levels (one is derived from the other). The error term (η) from the continuous block is assumed to follow a trivariate normal distribution and ω from the discrete block distributes logistically. To find the joint distribution, the transformation proposed by Lee (1983) was applied to the discrete choice part (Eq. (17)):

$$y_q = \Phi^{-1}(F(V_q)) \geq \Phi^{-1}(F(\omega_q)) = \omega_q^* \sim \mathcal{N}(0,1) \tag{21}$$

After this modification, the components from both blocks of the system are normally distributed and thus one can apply the "Conditional normal distribution" Theorem. These steps produce the following log-likelihood formula:

$$LL = \sum_i \sum_k \sum_q \alpha_{ikq}\delta_{ikq} \, ln\left(\phi(\eta_{1i})^{W_{1i}} \phi\left(\frac{\eta_{2i} - \mu_{\eta_{2i}|\eta_{1i}}}{\sigma_{\eta_2|\eta_1}}\right)^{W_{2i}} \phi\left(\frac{\eta_{3i} - \mu_{\eta_{3i}|\eta_{1i}\eta_{2i}}}{\sigma_{\eta_3|\eta_1\eta_2}}\right)^{W_{3i}} \Phi\left(\frac{y_{ikq} - \mu_{y_{ikq}|\eta_{1i}\eta_{2i}\eta_{3i}}}{\sigma_{y_q|\eta_1\eta_2\eta_3}}\right)\right) \tag{22}$$

where i indicates the person, k - the trip and q - the transport mode. α_{ikq} - is equal to zero if alternative q is not available for person i on trip k. and one otherwise. δ_{ikq} - is equal to zero if alternative q is not chosen for trip k of person i, and one otherwise. $\phi(.)$ and $\Phi(.)$ correspond to the density and distribution functions of the standard normal distribution. η_{mi} is the error term from the m-th continuous equation (Eq. (7)–(9)) for individual i. $\mu_{y|x}$ and $\sigma_{y|x}$ denote conditional mean and standard deviation. y_{ikq} is the Lee transformed probability (Eq. (17)) of mode q chosen by person i for trip k. Additionally, W_{1i}, W_{2i}, W_{3i} are weights applied only to the continuous equations. This might be used to balance the log-likelihood, if for one observation in the continuous block ($\{T_{w,i}, T_{f1,i}, E_{f1,i}, ...\}$) multiple choices/trips are available. The weights can be chosen to be proportional to the number of trips (n_i) made by each individual i.

The multinomial logit assumes "independence of irrelevant alternatives" (IIA) property, which in some cases might be doubtful.[5] Also, if a panel structure is present, choices across time might be correlated (Bhat and Gossen, 2004) or an unobserved individual-specific characteristic might affect the choice of travel mode (Toledo et al., 2007). To take the latter possibility into account, the formulation of the discrete model is updated. For this purpose, a normal error component model with latent variables (Walker et al., 2007) is used. This implies that the alternatives are correlated through the factor loadings (f_q) and the latent individual traits are expressed as factor ζ_i. The error term ϵ_q from Eq. (15) has the following form for alternative q and individual i:

$$\epsilon_{qi} = f_q \zeta_i + \nu_q \tag{23}$$

where ζ_i is a $(n_i \times 1)$ vector of i.i.d. standard normal variables (individual characteristics), f_q are mode-specific factor loadings, and ν_q is a vector of Gumbel distributed errors (Walker et al., 2007; Toledo et al., 2007).

2.3.1. Estimation

Coefficients belonging to the system of equations (Eqs. (7)–(9)) were divided by Θ. To estimate the joint continuous-discrete model accounting for the observed panel data structure, we employ hierarchical Bayesian (HB) estimation. In the first step, starting values for the Bayesian estimation were found by ML estimation. This was needed for faster and more stable convergence. To maximize the log-likelihood defined in Eq. (22), the R package *maxLik* (Henningsen and Toomet, 2011) was used. Optimisers from this package search for the local minima/maxima and use by default the numerical approximations of the gradient and the Hessian. With default settings, no convergence for our model was reached. Due to the complicated functional form (the likelihood function includes quantiles), the analytical gradient and the Hessian had to be computed by hand and later programmed into R. This improvement led to stable results. The initial starting values for ML estimation were defined for each block separately. The continuous block was estimated with ML and non-linear least squares were applied to the discrete one. Afterwards, both parts were optimized together using a combination of local optimizers ("BFGS" Fletcher, 1987, "NM" Nelder and Mead, 1965) and the evolutionary global optimization (Mullen et al., 2011) for fine tuning.[6]

After the starting values were found, hierarchical Bayesian estimation was employed using the R package RSGHB. The R code implementation for HB is based on Train and Sonnier (2005) and Train (2009). In the HB framework, all coefficients can be randomly distributed, but this is not always feasable. As Train (2009) points out, (i) random alternative-specific constants might be unidentifiable empirically; (ii) indicators (such as the VTTS) are ratios with more complex distributions than their elements and might result in economically unreasonable values (e.g. negative VTTS); (iii) the distribution of parameters might not be the main interest of the research. In our paper, we decided to keep all the coefficients fixed except for the individual-specific error components (ζ_i, Eq. (23)). In this setting, an individual makes multiple choices, which are assumed to be affected by unobservable personal characteristics (ζ_i) (Walker et al., 2007; Toledo et al., 2007). This accounts for the panel structure. The HB estimation was performed with 20,000 burn-ins and 40,000 iterations for averaging after the convergence has been reached. For more details on the estimation procedure, see Chapter 12.7.3 in Train (2009).

To sum up, the estimation proposed in Munizaga et al. (2008) was extended with three additions. First, an availability dummy was included to allow situations where not all alternatives are available. Second, multiple trips per individual were incorporated into the estimation by replicating the continuous part to match the number of trips and balancing the likelihood with individual-specific weights. Finally, the panel structure was modelled with the individual-specific error components.

[5] An alternative for this would be the usage of a multinomial probit model, but it would complicate the estimation procedure significantly, as multinomial probit does not have a closed solution with more than two alternatives. Another option would be to estimate mixed multinomial logit (MMNL) models. Schmid et al. (2019) has estimated a variety of logit modifications including the MMNL. Indeed, the MMNL improved the model fit, but the parameters did not change significantly. One could also apply the Copula method, which disassembles the joint multivariate density into a product of univariate densities and their Copula combinations. Bhat and Eluru (2009) presented a nice collection of bivariate Copulas and a good example of multivariate application is Sener et al. (2010).

[6] Although both algorithms, "BFGS" and "NM", are local optimisers, we found that the first tended to get stuck more often in local maxima, than the second one. To diminish the risk of staying in the local maximum, the estimation was fine tuned in three stages. First "BFGS" was used, than "NM" was applied with starting values from the previous step and finally the evolutionary global optimization was enforced.

3. Survey methods and data

In this paper, we use a novel data-set that distinguishes between 10 different activity types, 14 expenditure groups and 4 transport modes (walk, bike, car, public) over a period of one week, all of which were collected simultaneously, i.e., from the same individuals at the same time. The data was gathered using the newly developed Mobility-Activity-Expenditure Diary (MAED). It was conducted in the form of self-administered mail-back questionnaires with telephone support and incentives. The survey consisted of a diary and a household questionnaire. The diary had three sections: trip, activity and expenditure. In addition, infrequent long-term and regularly recurring payments were reported in the household questionnaire. As stated in Aschauer et al. (2019), this type of procedure is similar to consumer expenditure surveys, which gather retrospective information on long-term cost for one year. The survey took place in spring and autumn of 2015. The net sample included 748 representatively selected Austrian workers. The reporting period of one week was a compromise between response burden and accurate representation of the individuals' long-term equilibrium. Aside from the usual plausibility checks and error corrections, time-use and expenditure data were adjusted in order to reduce the incidental and unsystematic variation in the diary data and to better reflect the long-term equilibrium of individuals (Hössinger et al., 2019). To merge daily and long-term expenditure data, Hössinger et al. (2019) developed a three step procedure leading to reasonable results. The adjusted data is comparable with the Austrian Time Use Survey (ATU'S) 2008/09 and the Austrian Consumer Expenditure Survey (ACES) 2009/10 (Hössinger et al., 2019). The focus of this section is to give an overview of the model variables used in the estimation procedure. For a more detailed description of data, we refer to Aschauer et al. (2019), Aschauer et al. (2018) and Hössinger et al. (2019).

3.1. Time-use and expenditure data

The model defined by Eq. (7)–(9) requires the recorded data to be assigned to groups of freely chosen and committed activities/expenditures. Our time-use and expenditure categories are very broadly defined, so that everyone engages in each activity (no zeros in data). Thus, there is no need to accommodate for corner solutions that might arise with more detailed categorization. Table 1 shows the classification and shares of reported activity and expenditure categories into the model variables.

Although the influence of the classification on the results cannot be negated, the allocation is subjective as it cannot be validated. The used representation of committed activities (T_c[7]) is based on Jara-Díaz et al. (2016). The underlying logic is that most of the individuals do not want to spend more time than needed on domestic work, personal care, commuting or

Table 1
Shares and correlations of total expenditure and time-use data.

Activities	Var.	%	Expenditure	Var.	%	
	T_w	36.77		E_{f1}	17.26	
	T_{f1}	14.06		E_{f2}	5.73	
	T_{f2}	5.51		E_c	77.01	
	T_c	42.84				
Work	T_w	36.77	Leisure	E_{f1}	7.70	
Leisure	T_{f1}	14.06	Accomm	E_{f1}	5.95	
Eating	T_{f2}	4.52	Electronic	E_{f1}	3.61	
Shopping	T_{f2}	1.00	Clothes	E_{f2}	5.73	
Sleep	T_c	26.76	Housing	E_c	22.74	
Domestic	T_c	6.93	Food	E_c	17.46	
Personal	T_c	4.64	Mobility	E_c	12.47	
Travel	T_c	4.51	Insurance	E_c	7.83	
Education	T_c	0.63	Other	E_c	4.97	
Miscellaneous	T_c	0.20	Service	E_c	3.24	
			Health	E_c	2.55	
			Furniture	E_c	2.41	
			Education	E_c	2.06	
			Financing	E_c	1.29	
Correlations:						
	T_w	T_{f1}	T_{f2}	T_c	E_{f1}	E_{f2}
T_{f1}	−0.22***					
T_{f2}	−0.18***	−0.19***				
T_c	−0.60***	−0.58***	−0.04			
E_{f1}	0.39***	−0.12***	0.11**	−0.26***		
E_{f2}	0.26***	−0.03	0.12**	−0.23***	0.31***	
E_c	0.57***	−0.17***	−0.05	−0.33***	0.56***	0.36***

Signif. codes: *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$.

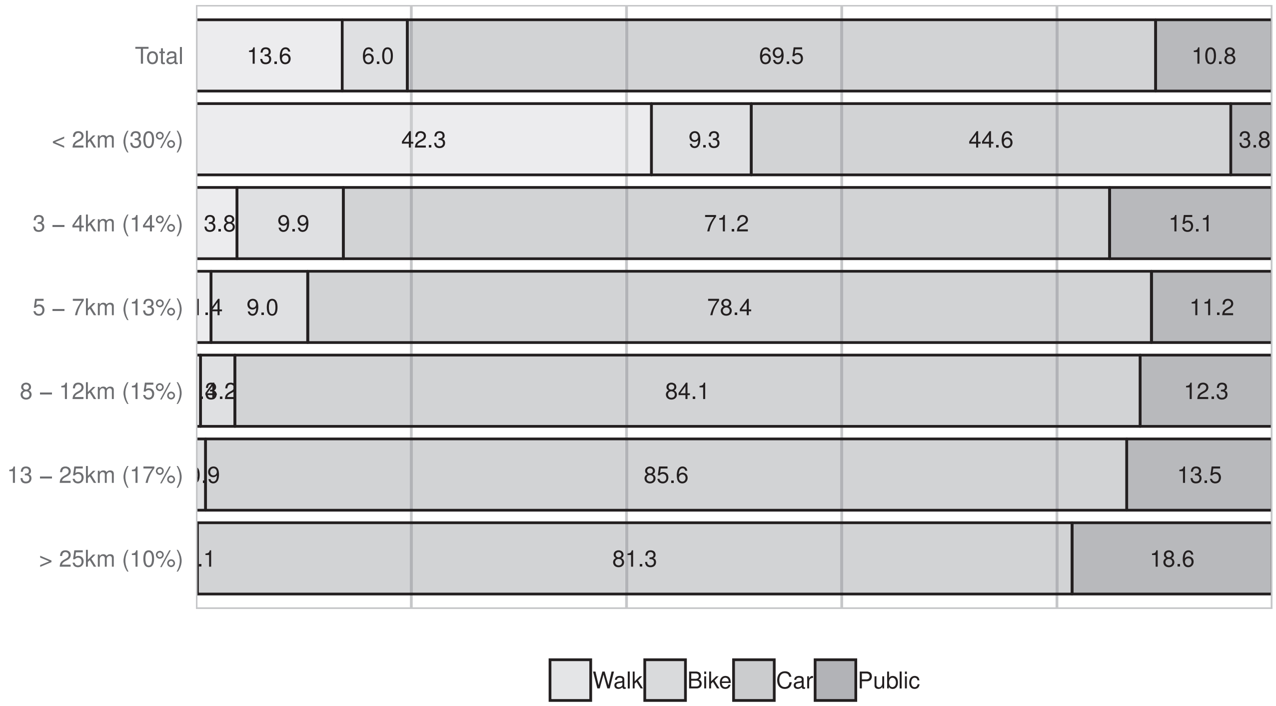

Fig. 1. Segmentation by the trip distance.

education. In contrast to Jara-Díaz et al. (2016) sleeping is considered to be a committed activity, as the individual minimum for biologically staying alive exists. The activity "Eating" was classified as an unrestricted one (T_{f2}[7]), as it includes eating in a restaurant and thus the necessary minimum time needed for food consumption may be exceeded.

The grouping of committed expenditures (E_c[7]) adopts the argumentation presented in Aschauer et al. (2019) as well as Mokhtarian and Chen (2004). Most importantly, people need to satisfy their basic needs (food, health, housing, education). Also, they do not want to spend too much money on household maintenance (Gronau and Hamermesh, 2006; Ahn et al., 2004) and transportation (Mokhtarian and Chen, 2004). Finally, some tasks simply have to be taken care of (mortgages, insurance). Due to their relaxed nature, expenditures on accommodation, leisure and recreational goods, as well as on electronics and communication devices were identified as freely chosen ones and grouped into E_{f1}[7]. Although clothing can be considered as necessity, expenses on it constituted a significant share of total spending (Table 1), evidently exceeding the "technical minimum" (for detailed information, see Hössinger et al., 2019).[7]

The observed individuals spend on average about 36.77% of their time working, and devote 77.01% of their income to committed activities (Table 1). All model variables are connected through time and budget constraints and thus changes in one variable will be reflected in the shift of another. The intra-continuous-block correlations are also presented in Table 1 and most of them are statistically significant. The joint estimation presented in Section 2.3 will take this into consideration.

3.2. Mobility

Due to the lack of mode choice data description in the previous studies (Hössinger et al., 2019; Schmid et al., 2019), a more thorough descriptive analysis of this part is presented here. The used data-set comprises 17,127 trips. In contrast to Jara-Díaz and Guevara (2003) and Munizaga et al. (2008), each individual had more than one trip and on average 23.24 trips were made per individual in the reporting week. The average length of a trip is 9.80 km and the average duration is 19.90 min. Fig. 1 shows the travel mode distribution in different segments of travel distance. In general, with the increase of travel distance, usage of car increases, reaching its peak in the segment "13 – 25 km". The only segment where the usage of cars drops drastically is "≤ 2 km" (but even then it still is used in 44.60% of the trips). This shortest distance segment corresponds to 30.22% of the total sample. In this segment people walked at least 10 times more often than in the other segments. What is more, the usage of public transport is highest in the segments "3 – 4 km" and "> 25 km". These are typical cases of intra-urban and inter-urban mobility.

[7] T_{f1} = {Leisure}, T_{f2} = {Eating, Shopping, Unspecified}, T_c = {Travel, Sleep, Education, Personal, Domestic, Other}, E_{f1} = {Leisure, Accommodation, Electronic}, E_{f2} = {Clothes}, E_c = {Housing, Food, Furniture, Health, Mobility, Education, Service, Financing, Insurance, Other}

In the discrete choice part (Eq. (17)), we have added the availability dummy α_q. Mode “Walk” is always available; mode “Public transport” is available if a public transportation route from the start to the end point exists; modes “Car” and “Bike” are considered unavailable if the individual does not own it. Mode “Bike” was available in 88.98% of the trips, “Car” in 91.95% and “Public transport” in 62.75%. Switching between the modes seems to not be that common, as 82.35% of the trips were done with the same mode as the previous one. The stickiest mode is “Car”, because in 91.50% (Table 2) of cases it remains the chosen mode. Also, the observed individuals usually switch to car, if they switch at all. If the previous trip was done with public transport, participants were more likely to walk than to use a car on the following trip.

Thus, even for the short distances individuals choose to go by car more often than to walk. This decision might be driven to a large extent by the duration of the trip. In 59.63% of cases, going by car was the quickest travel mode (Table 3). Only 35.29% of observed trips were carried out with a slower mode. The car was chosen even if walking would have been faster (74.68% of cases). Also, the socio-economic factors might influence mode choice. Respondents living in rural areas use the car twice as often as their urban counterparts (Appendix Fig. B.3). Persons without high school education and people with children tend to use a car more often and travel with public transport less often, compared to their counterparts. From this analysis, one can conclude that the trip duration is not the only factor influencing the travel mode choice. Thus, pre-commitments to modes via vehicle ownership, lifestyle, socio-economic status or comfort perception also play a role in the decision making.

Correlations between continuous variables and mode choice probabilities can be seen in Table 4. The individual probability of choosing a specific mode was defined as ratio between the frequency of choosing a specific mode and the total number of trips made. Although intra-block (within discrete/continuous block) correlations are high, inter-block (between mode choice and activities/expenditures) correlations are low. Munizaga et al. (2008) had estimated inter-correlations of up to 0.7, but in the MAED data-set the observed ones are only close to 0.1.

To estimate the probability of choosing a specific mode, one needs to specify the indirect utility function V_q (Eq. (15)). In this study, the following linear functional forms were assumed:

$$V_q = \alpha_q + \beta_{tq} time_q + \omega_q I_q + \gamma_{L,q} L_q time_q + \alpha_{L,q} L_q + \gamma_{W,q} W_q time_q + \alpha_{W,q} W_q + O_q \tag{24}$$

Table 2
Percentage of mode switching between successive trips, %.

$->$	Walk	Bike	Car	Public
Walk	57.76	3.37	23.89	14.98
Bike	8.22	71.54	15.53	4.71
Car	4.78	1.30	91.50	2.42
Public	20.03	2.48	16.99	60.50

Table 3
Distribution of slower chosen mode, in %.

Mode	Fastest	Not chosen, when fastest	Substituted by:			
			Walk	Bike	Car	Public
Total		35.29	17.63	16.39	54.19	11.79
Walk	8.30	22.53		23.70	74.68	1.62
Bike	0.63	78.85	60.98		24.39	14.63
Car	59.63	14.20	25.54	26.54		47.92
Public	31.43	77.79	15.35	12.65	72.00	

Table 4
Observed correlations between mode choice and time-use.

	P(Walk)	P(Bike)	P(Car)	P(Public)
P(Bike)	−0.01			
P(Car)	−0.63***	−0.41***		
P(Public)	0.16***	−0.06	−0.72***	
T_w	−0.08*	0.03	0.04	−0.01
T_{f1}	−0.07	−0.05	0.05	0.01
T_{f2}	0.08*	0.03	−0.03	−0.04
T_c	0.10**	0.01	−0.07	0.02
E_{f1}	0.07	0.08*	−0.10**	0.04
E_{f2}	−0.05	0.03	0.04	−0.04
E_c	−0.06	0.02	0.06	−0.07

Signif. codes: *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$.

$$O_q = \begin{cases} -\alpha_q - \alpha_{L,q} - \alpha_{W,q}, \text{ if q} = 1(\text{Walk}) \\ \emptyset, \text{ if q} = 2(\text{Bike}) \\ \beta_{cost} cost_3 + \beta_{PH} HhPark_3 + \beta_{JP} JobPark_3 + \beta_{MGP} \text{MgPark}_3, \text{ if q} = 3(\text{Car}) \\ \beta_{cost} cost_4 + \beta_{t2b} t2bus_4 + \beta_{svI} servInt_4 + \beta_{stp} stops_4, \text{ if q } = 4(\text{Public Transport}) \end{cases} \quad (25)$$

The index set {1, 2, 3, 4} corresponds to mode set {Walk, Bike, Car, Public transport}. Variable $time_q$ represents the duration of a trip with mode q, and $cost_q$ is the cost of a trip with mode q. To incorporate "stickiness" to a particular mode, as shown in Table 2, the inertia variable I_q was created. Following the approach from Börjesson et al. (2013), Cherchi et al. (2013), and Schmid et al. (2019) the inertia variable is a dummy, which is equal to one if the mode chosen by a person for a trip at the start of the current tour is the same as the one chosen in the previous tour made for the same purpose, and zero otherwise. To account for different trip purposes, variables L_q (leisure) and W_q (work/education) were incorporated into the estimation framework. They were created using effect coding and their effects ($\gamma_{L,q}$, $\gamma_{W,q}$) can thus be interpreted as deviations from the average. $HhPark_3$ is a dummy for parking availability at home, $JobPark_3$ - a dummy for parking availability at work place, $MgPark_3$ - a dummy for parking management in-force of the destination of the trip, $t2bus_4$ - the access time (time from start to the first station; time to destination from the last station), $servInt_4$ - public transport service interval in minutes, $stops_4$ - the necessary number of changes to reach the destination with public transport. The latter variable is equal to zero for trips outside Vienna.

4. Results

Four models were estimated with the estimation procedure described in Section 2. The first model ("w/o corr") corresponds to the model without inter continuous-discrete block correlations, while the second model includes them ("w/ corr"). The third model ("w/o E_{f1}") was estimated without expenditure equation (Eq. (9)), but with inter-block correlations. The last model ("w/o Panel") was estimated with inter continuous-discrete block correlations, but does not account for the panel structure. As mentioned before, there is a disbalance between the number of observations in continuous and discrete data: individual i has only one set of $\{T_w, T_{f1}, E_{f1}, \ldots\}$ and multiple trips (n_i). To merge these parts, $\{T_w, T_{f1}, E_{f1}, \ldots\}$ observations were replicated n_i times for individual i. This could cause a bias in the likelihood and the estimation results. To correct for these potential distortions, the weights ($W_1 = W_2 = W_3$) were applied to the continuous equations. They were chosen to be indirectly proportional to the number of trips made by individual i ($W_{i1} = 1/n_i$).

The estimation results are shown in Table 5. All models are unique to this study as they present for the first time outcomes from the joint time-use and mode choice model, while accounting for the panel structure of the underlying data (except model "w/o Panel"), and including an expenditure equation (except model "w/o E_{f1}"). The third model ("w/o E_{f1}") serves as a reference for comparability with the earlier works, as it is similar to the one used in Jara-Díaz and Guevara (2003) and Munizaga et al. (2008), both of which do not include an exogenous expenditure modelling procedure.

The analysis begins with a comparison of the models without inter-continuous-discrete block correlations ("w/o corr") and with ("w/ corr"). There is a considerable likelihood improvement and the McFadden R^2 (ρ^2) is higher, if inter-block correlations are considered. This is caused by the estimation of possible correlations, as well as the changes in the values of parameters. As noted in Munizaga et al. (2008), these correlations have to be interpreted with the opposite sign. Thus, the negative value of $\rho_{Tw\&car}$ is an indicator of unobservable factors that make people assign more time to work and, simultaneously, have a higher propensity to use the car. Also, $\rho_{\text{Ef1\&car}}$ is negative, reflecting the possibly higher expenditures of car drivers. The lowest absolute correlations are between work time (T_w) and three transportation modes: public transport, walk, and bike. In general, work time and uncommitted expenditures (group E_{f1}) have mostly negative correlations with all transport modes and free time activity (group T_{f1}) has positive correlations. Also, car made has the highest absolute correlations with the continuous equations. This might be due to the fact that the MAED sample is strongly dominated by car users since about 70% of all trips were made by car. The exclusion of these relationships (inter-block correlations) results in a 17.04% underestimation of the VoL.

In previous studies, the joint time-use and activity model was estimated without an expenditure equation. To investigate the effect of incorporating expenditures, we have estimated a model without expenditure equation (Column "w/o E_{f1}"). The likelihoods of the "w/o E_{f1}" and of the other models are not comparable, as this model includes fewer data points to be estimated and thus produces fewer errors. The biggest difference in the estimates appears to be the value of elasticity of utility with respect to work time (θw). It seems that the "w/o E_{f1}" model transfers some baseline utility from goods to work, as Φ becomes smaller and θ_w increases, but remains negative. From Equation (13) from Jara-Díaz et al. (2008) ($(\theta_w U)/(T_w) + \lambda w - \mu = 0$) it is clear that, if $\theta_w \to 0$, then $VoL = \mu/\lambda = w$. Thus, the value of leisure equals the wage rate (w) and one falls back to the assumption made in Train and McFadden (1978) and Becker (1965). This can be seen in the VoL from the "w/o E_{F1}" model. If expenditures are ignored, the difference between the wage rate and the VoL diminishes to around 1€/h and the VoL is overestimated by 16.83%. What is more, consideration of the panel structure according to Eq. (23) plays a role. It improves the overall model fit from 0.608 ("w/o Panel") to 0.634 ("w/" corr) and represents the observed situation (repeated observations) better. We conclude that the model with expenditures and panel structure better reflects the real preferences of the individuals and thus we use it for further analysis.

In the final model, 57 parameters were estimated, of which 10 belong to the continuous block. Parameter θ_w is negative, indicating that work generates disutility. This is also confirmed by the negative value of VTAW. On average, the disutility of

Table 5
Estimation results.

	w/o corr Par [s.d.]	w/ corr Par [s.d.]	w/o E_{f1} Par [s.d.]	w/o Panel Par' [s.d.]
Activities models parameters				
θ_w	−0.504 [0.040]	−0.256 [0.053]	−0.072 [0.016]	−0.318 [0.021]
Φ	0.391 [0.015]	0.296 [0.019]	0.255 [0.009]	0.319 [0.012]
θ_1	0.732 [0.004]	0.744 [0.007]	0.738 [0.005]	0.745 [0.004]
ϕ_1	0.242 [0.009]	0.174 [0.012]		0.182 [0.006]
Mode constants				
α_{bike}	−3.350 [0.040]	−3.200 [0.035]	−3.170 [0.034]	−3.070 [0.029]
α_{car}	−1.950 [0.046]	−1.980 [0.034]	−1.970 [0.029]	−2.030 [0.017]
α_{PT}	−2.120 [0.059]	−2.190 [0.098]	−2.000 [0.020]	−2.020 [0.020]
Time parameters				
β_{walk}	−0.171 [0.005]	−0.171 [0.008]	−0.168 [0.005]	−0.164 [0.004]
β_{bike}	−0.091 [0.005]	−0.090 [0.007]	−0.087 [0.005]	−0.064 [0.003]
β_{car}	−0.096 [0.008]	−0.098 [0.014]	−0.086 [0.008]	−0.082 [0.006]
β_{PT}	−0.041 [0.005]	−0.038 [0.007]	−0.040 [0.006]	−0.034 [0.004]
Mode choice taste parameters				
β_{t2bus}	−0.059 [0.008]	−0.058 [0.013]	−0.060 [0.008]	−0.051 [0.006]
β_{cost}	−0.653 [0.036]	−0.591 [0.057]	−0.602 [0.013]	−0.484 [0.020]
$\beta_{servInt}$	−0.028 [0.003]	−0.028 [0.005]	−0.028 [0.004]	−0.026 [0.003]
β_{stops}	−0.243 [0.040]	−0.423 [0.063]	−0.154 [0.021]	−0.227 [0.018]
β_{HhPark}	0.538 [0.035]	0.586 [0.056]	0.386 [0.022]	0.474 [0.037]
$\beta_{JobPark}$	0.612 [0.016]	0.630 [0.032]	0.605 [0.034]	0.503 [0.037]
β_{MgPark}	−1.170 [0.064]	−1.200 [0.098]	−0.955 [0.043]	−0.941 [0.029]
Inertia				
ω_{walk}	2.590 [0.030]	2.680 [0.028]	2.650 [0.016]	2.780 [0.030]
ω_{bike}	4.140 [0.092]	4.190 [0.032]	4.360 [0.020]	4.240 [0.030]
ω_{car}	2.450 [0.049]	2.260 [0.029]	2.340 [0.037]	2.170 [0.021]
ω_{PT}	1.820 [0.031]	1.890 [0.022]	1.740 [0.017]	1.870 [0.021]
Trip purpose: leisure x time				
$\gamma_{L,\ walk}$	0.060 [0.007]	0.059 [0.011]	0.055 [0.007]	0.060 [0.005]
$\gamma_{L,bike}$	−0.008 [0.006]	0.001 [0.009]	−0.006 [0.006]	0.004 [0.006]
$\gamma_{L,car}$	0.001 [0.011]	0.011 [0.015]	0.010 [0.012]	0.018 [0.010]
$\gamma_{L,PT}$	−0.014 [0.009]	−0.007 [0.012]	−0.015 [0.010]	−0.005 [0.008]
Trip purpose: leisure				
$\alpha_{L,bike}$	0.709 [0.035]	0.433 [0.027]	0.602 [0.022]	0.479 [0.039]
$\alpha_{L,car}$	0.621 [0.027]	0.469 [0.031]	0.404 [0.032]	0.421 [0.020]
$\alpha_{L,PT}$	0.699 [0.029]	0.638 [0.036]	0.728 [0.024]	0.732 [0.024]
Trip purpose: work x time				
$\gamma_{W,\ walk}$	−0.086 [0.008]	−0.089 [0.014]	−0.077 [0.008]	−0.085 [0.008]
$\gamma_{W,bike}$	0.008 [0.005]	0.004 [0.007]	0.009 [0.007]	−0.002 [0.005]
$\gamma_{W,car}$	0.014 [0.009]	0.000 [0.016]	−0.001 [0.012]	−0.002 [0.010]
$\gamma_{W,PT}$	0.010 [0.007]	0.001 [0.011]	0.008 [0.009]	0.002 [0.007]
Trip purpose: work				
$\alpha_{W,bike}$	−1.060 [0.026]	−1.050 [0.052]	−1.040 [0.058]	−0.923 [0.029]
$\alpha_{W,car}$	−1.370 [0.073]	−1.190 [0.051]	−1.020 [0.023]	−1.160 [0.036]
$\alpha_{W,PT}$	−1.060 [0.031]	−0.973 [0.025]	−0.976 [0.020]	−1.010 [0.065]
Factor loadings				
f_{bike}	−1.030 [0.078]	−0.916 [0.034]	−0.798 [0.018]	
f_{PT}	−0.365 [0.050]	−0.411 [0.028]	−0.239 [0.024]	
f_{car}	1.160 [0.064]	1.200 [0.067]	1.260 [0.018]	
Standard deviations				
$\hat{\sigma}_{Tw}$	63.000 [0.505]	61.500 [0.361]	58.700 [0.345]	60.900 [0.389]
$\hat{\sigma}_{Tf1}$	67.500 [0.847]	64.700 [0.193]	66.900 [0.208]	64.700 [0.153]
$\hat{\sigma}_{Ef1}$	42.200 [0.726]	36.500 [0.626]		35.500 [0.436]
Correlations (activities)				
$\rho_{Tw\&Tf1}$	−0.695 [0.016]	−0.702 [0.015]	−0.697 [0.016]	−0.708 [0.014]
$\rho_{Tw\&Ef1}$	0.350 [0.031]	0.405 [0.029]		0.421 [0.025]
$\rho_{Tf1\&Ef1}$	−0.462 [0.027]	−0.435 [0.027]		−0.411 [0.028]
Correlations (discrete/continuous)				
$\rho_{Tw\&walk}$		−0.065 [0.025]	−0.044 [0.024]	−0.071 [0.022]
$\rho_{Tw\&bike}$		−0.104 [0.030]	−0.082 [0.027]	−0.086 [0.024]
$\rho_{Tw\&PT}$		0.028 [0.031]	0.081 [0.030]	0.053 [0.028]
$\rho_{Tw\&car}$		−0.181 [0.038]	−0.077 [0.030]	−0.150 [0.032]
$\rho_{Tf1\&walk}$		0.223 [0.024]	0.217 [0.023]	0.220 [0.022]
$\rho_{Tf1\&bike}$		0.207 [0.028]	0.193 [0.027]	0.201 [0.025]
$\rho_{Tf1\&PT}$		0.151 [0.031]	0.135 [0.031]	0.121 [0.028]
$\rho_{Tf1\&car}$		0.275 [0.035]	0.310 [0.027]	0.207 [0.031]

(*continued on next page*)

Table 5 (*continued*)

	w/o corr Par [s.d.]	w/ corr Par [s.d.]	w/o E_{f1} Par [s.d.]	w/o Panel Par‘ [s.d.]
$\rho_{Ef1\&walk}$		−0.327 [0.021]		−0.354 [0.019]
$\rho_{Ef1\&bike}$		−0.408 [0.024]		−0.394 [0.020]
$\rho_{Ef1\&PT}$		−0.470 [0.025]		−0.465 [0.022]
$\rho_{Ef1\&car}$		−0.585 [0.021]		−0.560 [0.020]
Value of time				
wage	12.14			
VoL	7.708 [3.278]	9.291 [3.896]	11.172 [4.639]	8.799 [3.681]
VTAW	−4.428 [2.552]	−2.845 [1.829]	−0.964 [0.664]	−3.337 [2.027]
$VTTS_{walk}$: *Total*	15.738 [0.852]	17.525 [1.612]	16.723 [0.690]	20.342 [0.997]
work	23.611 [1.240]	26.552 [2.484]	24.435 [1.471]	30.861 [2.101]
leisure	10.222 [0.972]	11.478 [1.613]	11.192 [0.753]	12.854 [0.780]
other	13.381 [0.798]	14.546 [1.460]	14.543 [0.591]	17.312 [0.801]
$VTTS_{bike}$: *Total*	8.380 [0.447]	9.245 [0.785]	8.697 [0.523]	7.918 [0.429]
work	7.642 [0.563]	8.794 [1.058]	7.854 [0.970]	8.165 [0.852]
leisure	9.162 [0.863]	9.180 [1.285]	9.271 [0.865]	7.440 [0.768]
other	8.335 [0.482]	9.762 [0.899]	8.966 [0.526]	8.150 [0.505]
$VTTS_{car}$: *Total*	8.837 [0.769]	9.978 [1.352]	8.616 [0.844]	10.153 [0.809]
work	7.499 [1.134]	9.956 [2.359]	8.773 [1.642]	10.363 [1.736]
leisure	8.792 [1.467]	8.866 [1.850]	7.593 [1.474]	7.868 [1.342]
other	10.219 [0.804]	11.112 [1.294]	9.481 [0.962]	12.229 [1.057]
$VTTS_{PT}$: *Total*	3.775 [0.502]	3.908 [0.748]	3.978 [0.587]	4.225 [0.470]
work	2.808 [0.733]	3.766 [1.457]	3.233 [1.154]	3.952 [1.006]
leisure	5.038 [1.177]	4.703 [1.430]	5.454 [1.235]	4.818 [1.114]
other	3.479 [0.490]	3.254 [0.770]	3.247 [0.621]	3.906 [0.562]
$VTAT_{walk}$: *Total*	−8.030 [3.364]	−8.234 [4.080]	−5.552 [4.678]	−11.543 [3.817]
work	−15.902 [3.485]	−17.260 [4.481]	−13.263 [4.836]	−22.062 [4.253]
leisure	−2.514 [3.391]	−2.187 [4.083]	−0.020 [4.700]	−4.054 [3.761]
other	−5.673 [3.355]	−5.255 [4.037]	−3.371 [4.671]	−8.513 [3.764]
$VTAT_{bike}$: *Total*	−0.672 [3.301]	0.046 [3.918]	2.474 [4.660]	0.881 [3.707]
work	0.066 [3.328]	0.497 [4.012]	3.318 [4.718]	0.634 [3.788]
leisure	−1.454 [3.374]	0.111 [4.012]	1.900 [4.717]	1.359 [3.754]
other	−0.627 [3.306]	−0.470 [3.949]	2.205 [4.667]	0.649 [3.716]
$VTAT_{car}$: *Total*	−1.129 [3.357]	−0.687 [4.070]	2.556 [4.702]	−1.354 [3.776]
work	0.209 [3.460]	−0.665 [4.547]	2.398 [4.881]	−1.563 [4.098]
leisure	−1.084 [3.568]	0.426 [4.215]	3.578 [4.871]	0.931 [3.913]
other	−2.511 [3.379]	−1.821 [4.052]	1.691 [4.735]	−3.429 [3.825]
$VTAT_{PT}$: *Total*	3.933 [3.309]	5.384 [3.937]	7.194 [4.670]	4.574 [3.710]
work	4.900 [3.360]	5.525 [4.181]	7.938 [4.752]	4.848 [3.829]
leisure	2.670 [3.460]	4.588 [4.056]	5.718 [4.809]	3.981 [3.833]
other	4.229 [3.316]	6.038 [3.958]	7.925 [4.682]	4.894 [3.721]
Goodness of fit				
LL_{null}	−35129.857	−36315.319	−31826.792	−36315.319
LL_{model}	−14065.252	−13284.852	−10862.591	−14250.575
ρ^2	0.600	0.634	0.659	0.608
AIC	29678.505	28117.704	23271.182	30049.150
#parameters	45	57	49	54

work is estimated to be around 2.8€/h. Parameters Φ (sum of all exponents of freely chosen goods), ϕ_1 (the first group of freely chosen goods) and θ_1 (the first group of freely chosen activities) positively effect utility. As not all modes are always available, the four mode constants do not represent the market shares in the sample. Time and cost parameters represent the negative marginal utility of having to pay or to spend time on travelling. Public transport parameters β_{t2bus}, $\beta_{servInt}$ and β_{stops} depict the displeasure in having to walk to/from a station, to wait more for the next bus, and to change transport more often, respectively. Having parking place near home (β_{HhPark}) or work ($\beta_{JobPark}$) has a positive effect on choosing car as transport mode, and, conversely, the presence of parking pricing scheme (β_{MgPark}) has a negative effect. As it was expected, all inertia effects have positive signs, indicating positive effects of preferences in previous tours on the current one with the same trip purpose.

In comparison to the other models with panel structure, the cost coefficient (β_{cost}) from the Model "w/ corr" is smaller in absolute terms, but still negative. This results in higher estimates of the VTTS for all modes. All models display the same ordering (from high to low) of the VTTS: walk, car, bike, public transport. This finding is consistent with Schmid et al. (2019). Interestingly, car exhibits a higher VTTS than public transport. The latter has the highest VTTS for leisure trips, whereas the VTTS of "Car" is highest for the "other" purpose trips. Overall, the highest VTTS is observed for work-related trips by mode "Walk" (26€/h, which is double the average wage rate), indicating unwillingness to walk to work. The binding link between VTTS and VTAT predetermines the negative relationship between them. The low VTTS of public transport is caused by the positive and significant VTAT, which captures the good public transport conditions in Austria, and might explain the different ordering of alternatives in comparison to studies from other countries. The VTAT for car is smaller than for PT and negative, which contradicts the general belief, that travelling by car is more pleasant. Better travel conditions of public transport might be caused by the possibility to engage in secondary activities (listen to music, read, surf the web, etc.) or by the lifted burden of driving and spending less time in traffic jams. To conclude the final model (with inter-block correlations and panel structure) is the most informative, as it allows to take into account most of the available information. Therefore, it is used in the following segmentation analysis of the value of time indicators, VoL, VTTS and VTAT.

4.1. Segmentation

For the sake of comparability with the previous studies based on the same dataset (Schmid et al., 2019; Hössinger et al., 2019), the sample was divided by urbanity, gender, age, education, parenthood, number of workers in the household and personal income, all of which are expressed as dummy variables representing a “lower” and “upper” group. A priori segmentation was applied to the data and afterwards the model proposed in Eq. (22) was estimated. The results of the 14 different models are presented in Fig. 2. There are considerable differences between some of the segments.

The largest absolute intra-segment VoL differences are observed for the following partitions: “Pers. income” (9.82€/h), “No. of workers” (6.32€/h), “Gender” (4.76€/h) and “Age” (4.05€/h). Hössinger et al. (2019) discuss the possible reasons for that. The study tries to explain the potential relationship between Φ (used in the VoL calculation) and the variance of T_w, arguing that “a high variance causes a low VoL and vice versa”. As an example, the segmentation by gender. Male respondents have high values of observed T_w with a low variance in working time and females have lower values of T_w with a higher variance in working time (mainly due to part-time work being more common for females). The same can be said about single workers, who are working mainly full time and thus have low variance in T_w. An additional reason for these disparities are the considerable differences in working time and time assigned to domestic work. In the MAED sample, women spend close to 9 h/week less in the paid work and around the same amount more in the domestic work.

Another crucial part for the derivation of the VTAT is the VTTS. Despite the similarity in the mode-specific VTTS ranking to other recent valuation studies (Schmid et al., 2019; Börjesson and Eliasson, 2014; Kouwenhoven et al., 2014; Axhausen et al., 2014; Fröhlich et al., 2012; Weis et al., 2012), it is more profoundly expressed in the current study. Fig. 3 gives an overview of all time indicators in different segments. Participants selecting transport modes “Bike” and “Car” exhibit similar willingness to pay for additional unit of leisure and reduction of travel time, whereas the VTTS and VoL for both “Walk” and “PT” differ considerably.

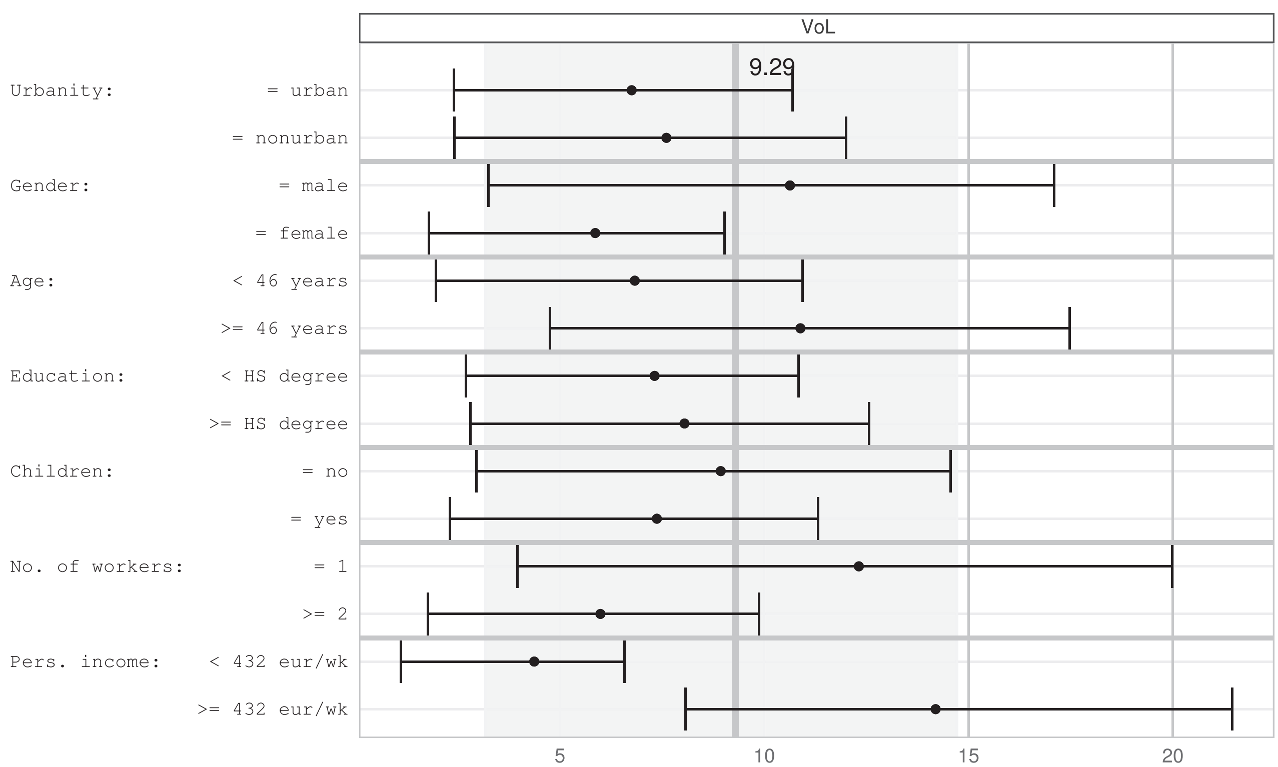

Fig. 2. VoL analysis, grey area represents 95% credible region.

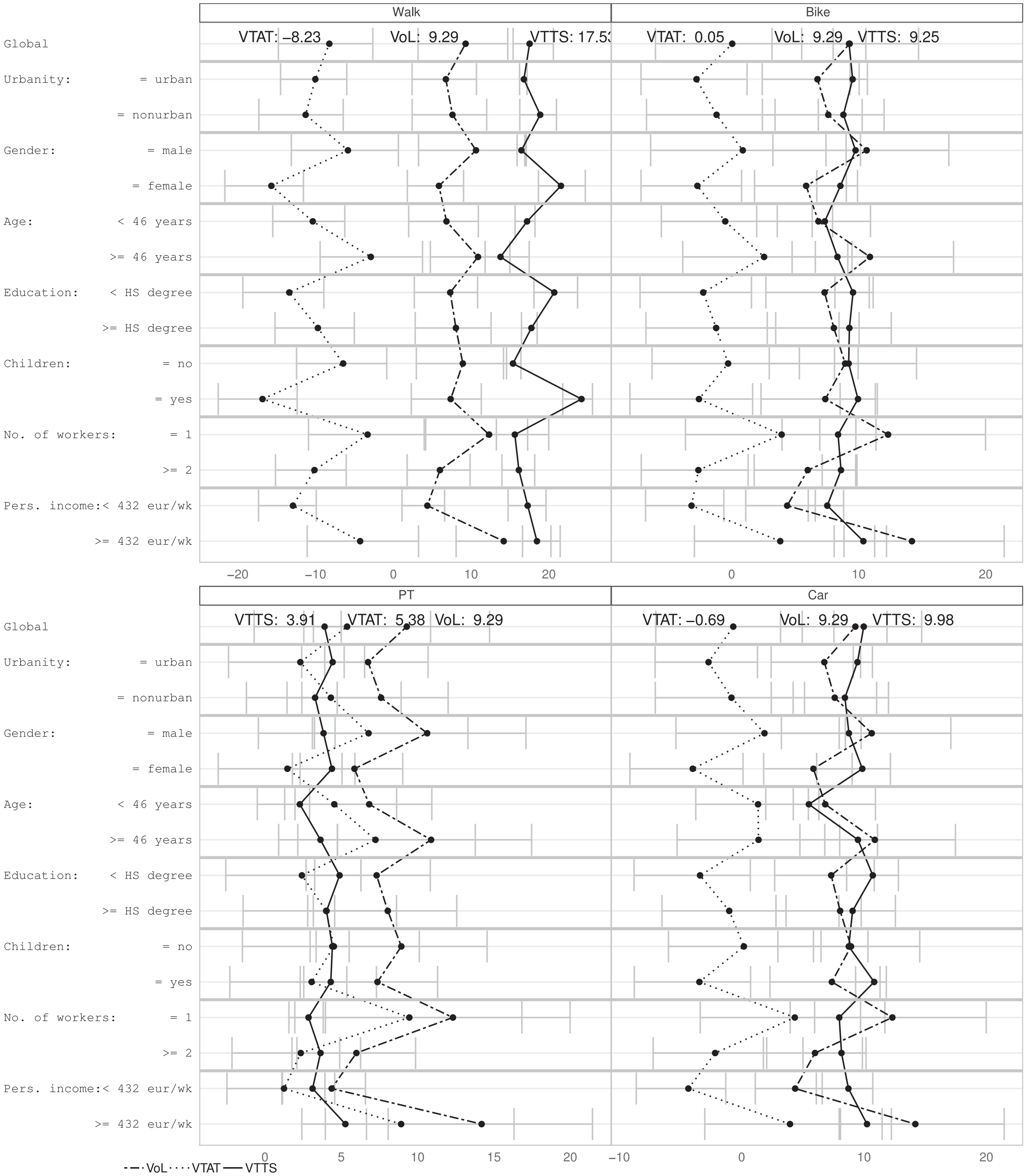

Fig. 3. Mode specific indicators.

Further analysis concentrates on the differences between car and public transport (PT), as the Austrian infrastructure expenditure on these modes make up a substantially larger share than on walking or cycling. Differences between VTTS for car and public transport can be seen in Table 6. The average difference in the VTTS is estimated to be around 6.07€/h and in the study of Schmid et al. (2019)[8] - around 4€/h. To explain the disparity in the willingness to pay to reduce travel time,

[8] Schmid et al. (2019) use stated and revealed preference data to estimate the models. This partly explains why the estimated VTTS of public transport differs considerably from the current study, which used only the revealed preference data.

Table 6
Mode and user type effects. Here $ME = \overline{ME}_{car-PT}$ and $UE = \overline{UE}_{1-0}$.

	Purpose	ΔVTTS	ME	UE	UE_{VoL}	UE_{VTAT}	Δ $VTAT_{car}$	Δ $VTAT_{PT}$	AIC
Global	Total	6.07							28117.70
Urbanity	Total		5.12	−1.04	0.85	1.89	1.88	2.00	28585.56
Gender	Total		5.20	1.02	−4.76	−5.79	−5.86	−5.32	28277.70
Age	Total		4.57	3.66	4.05	0.40	0.04	2.70	28029.54
Education	Total		5.34	−1.55	0.73	2.28	2.39	1.59	28344.98
Children	Total		5.13	1.78	−1.56	−3.34	−3.64	−1.46	28114.69
No. of workers	Total		4.64	0.27	−6.32	−6.59	−6.51	−7.11	28034.97
Pers. income	Total		5.29	1.60	9.82	8.22	8.30	7.67	27786.66
Global	Work	6.19							28117.70
Urbanity	Work		4.83	4.04	0.85	−3.19	−3.53	−1.00	28585.56
Gender	Work		5.98	2.40	−4.76	−7.16	−7.55	−4.65	28277.70
Age	Work		4.77	3.28	4.05	0.78	0.28	3.96	28029.54
Education	Work		5.34	−2.55	0.73	3.28	3.69	0.70	28344.98
Children	Work		5.74	6.80	−1.56	−8.36	−8.91	−4.82	28114.69
No. of workers	Work		4.85	−1.59	−6.32	−4.73	−4.64	−5.32	28034.97
Pers. income	Work		5.24	−0.30	9.82	10.13	10.30	9.01	27786.66
Global	Leisure	4.16							28117.70
Urbanity	Leisure		2.24	−9.47	0.85	10.32	11.17	4.84	28585.56
Gender	Leisure		1.50	−0.14	−4.76	−4.62	−4.27	−6.87	28277.70
Age	Leisure		2.31	6.06	4.05	−2.01	−2.36	0.25	28029.54
Education	Leisure		3.23	−0.83	0.73	1.56	1.41	2.53	28344.98
Children	Leisure		1.76	−6.75	−1.56	5.18	5.48	3.26	28114.69
No. of workers	Leisure		2.00	−1.19	−6.32	−5.13	−4.64	−8.30	28034.97
Pers. income	Leisure		3.12	3.34	9.82	6.48	6.39	7.10	27786.66
Global	Other	7.86							28117.70
Urbanity	Other		8.31	2.30	0.85	−1.45	−2.01	2.16	28585.56
Gender	Other		8.11	0.81	−4.76	−5.58	−5.76	−4.43	28277.70
Age	Other		6.64	1.63	4.05	2.42	2.19	3.89	28029.54
Education	Other		7.45	−1.26	0.73	1.99	2.06	1.53	28344.98
Children	Other		7.91	5.29	−1.56	−6.85	−7.48	−2.82	28114.69
No. of workers	Other		7.08	3.58	−6.32	−9.90	−10.25	−7.71	28034.97
Pers. income	Other		7.52	1.78	9.82	8.05	8.22	6.91	27786.66

Schmid et al. (2019) followed the approach proposed by Flügel (2014), which divides the VTTS into two parts: the mode effect (ME) and the user-type effect (UE). The pure average ME is based on the weighted average of the differences in the VTTS between car and PT within each user group. It can also be expressed as the weighted average of differences in the VTAT:

$$\begin{aligned} ME = \overline{ME}_{car-PT} &= \frac{N_0(VTTS_{car,0} - VTTS_{PT,0}) + N_1(VTTS_{car,1} - VTTS_{PT,1})}{N_0 + N_1} \\ &= \frac{N_0(VTAT_{PT,0} - VTAT_{Car,0}) + N_1(VTAT_{PT,1} - VTAT_{Car,1})}{N_0 + N_1} \end{aligned} \tag{26}$$

Here, the first user group is denoted by 0 and the second by 1. N_0 is the number of users in group 0 and N_1 in group 1. If a user type is controlled by some variable (e.g. by including the interaction term), lower values of $\overline{ME}_{car-PT}$ will indicate higher explanatory power of the grouping variable in explaining $\Delta VTTS$.

The user type effect ($\overline{UE}_{1-0}$) is defined as the VTTS differences between the two user-groups within each mode and weighted according to the number of observed choices of PT(N_{PT}) and car(N_{car}). The joint estimation framework allows for further decomposition of UE. Using the relationship $VTTS = VTAT - VoL$, the UE can be additionally disentangled into UE_{VoL} and UE_{VTAT}. This enables to explain the UE through the differences in perception of leisure and time assigned to travel.

$$\begin{aligned} UE = \overline{UE}_{1-0} &= \frac{N_{car}(VTTS_{car,1} - VTTS_{car,0}) + N_{PT}(VTTS_{PT,1} - VTTS_{PT,0})}{N_{car} + N_{PT}} \\ &= \frac{N_{car}(VoL_1 - VTAT_{car,1} - VoL_0 + VTAT_{car,0}) + N_{PT}(VoL_1 - VTAT_{PT,1} - VoL_0 + VTAT_{PT,0})}{N_{car} + N_{PT}} = \\ &= \underbrace{(VoL_1 - VoL_0)}_{UE_{VoL}} - \underbrace{\frac{N_{car}(VTAT_{car,1} - VTAT_{car,0}) + N_{PT}(VTAT_{PT,1} - VTAT_{PT,0})}{N_{car} + N_{PT}}}_{UE_{VTAT}} \end{aligned} \tag{27}$$

Here, the $\overline{UE}_{1-0}$ is decomposed into the UE_{VoL} (differences in leisure perception within each segment) and the differences between the weighted averages of user-type-specific VTAT (value of time spent while travelling). This new decomposition can be calculated due to the joint modelling framework presented in this study, with simultaneous computation of VoL and VTAT. As mentioned before, the difference in the VTTS between car and public transport for the average trip is estimated to be 6.07€/h. The proposed decomposition into user and mode type effect was used to disentangle this difference.

If the $\overline{UE}_{1-0}$ is 0, the *UE* of VoL and VTAT are equal (segments: “No. of workers” Table 6). In other words, the *UE* of both activities, leisure or travel, is the same. If it is positive and both UE_{VoL} and UE_{VTAT} are positive, there are bigger dissimilarities between the groups in the perception of leisure than in travel time (“Age”, “Pers. income”). If $\overline{UE}_{1-0} > 0$ and both UE_{VoL} and UE_{VTAT} are negative, bigger dissimilarities between groups in the perception of travel time than of leisure are present (segment: “Gender”, “Children”). If $\overline{UE}_{1-0} < 0$ and both $UE_{VoL} < 0$ and $UE_{VTAT} < 0$, groups are more heterogeneous in the valuation of leisure than towards travel time. If $\overline{UE}_{1-0} < 0$ and both $UE_{VoL} > 0$ and $UE_{VTAT} > 0$, groups are more heterogeneous in the attitude towards travel time than towards leisure (segment: “Urbanity”, “Education”). The MAED sample is strongly dominated by car travelers, as 69.54% of all trips were made by car and only 10.83% by public transport. Thus if car travelers of both segments perceive the travel time similarly ($\Delta VTAT_{Car}$ is small, segment “Age”), the UE is dominated by differences in leisure preferences

All segment-specific *ME* and *UE* values can be found in Table 6. In most of the segments and trip purposes, the mode effect is more profound than the user type effect and close to the global difference in the VTTS associated with car and public transport. Only for leisure-related trips, the user effect becomes dominant (segments: “Urbanity”, “Age”, “Children”) with more profound or negative differences in the perception of travel time (UE_{VTAT}).

The results indicate that the difference of 6.07€/h in the VTTS between car and public transport can be marginally reduced if the user effect is taken into account. In contrast to most of the other European studies on the VTTS, the user effect was found to be much smaller than the mode effect. Segmentation by “Age” exhibits the strongest power to disentangle the average VTTS difference between car and PT. This segmentation is also associated with the highest heterogeneity in the average VTTS independent of the mode (*UE*), which is driven by the differences in the perception of leisure (UE_{VoL}). All in all, the mode effect almost always dominates the user type effect. Higher values of the user effect are caused by more profound differences in the VoL than in the VTAT.

5. Synthesis and conclusions

The main objective of this study was to develop an advanced estimation procedure which facilitates the joint estimation of the discrete-continuous model framework with all its components (including time-use, expenditures, and each of the weekly travel choices) as proposed by Jara-Díaz and Guevara (2003), allowing individuals to make multiple trips and to estimate the parameters of this model framework with the MAED dataset in order to obtain the value of leisure, travel time savings, and time assigned to travel. Expenditures were obtained from the same individuals (not imputed) and all travel choices were considered simultaneously. The original framework was extended to incorporate multiple trips per individual, transport mode availability, weighting of likelihood, and to take into account the observed panel data structure.

The estimated values of time show that the average VoL is 9.29€/h and that the VTTS varies strongly between the modes (9.98€/h for car, 3.91€/h for public transport, 9.25€/h for bike and 17.53€/h for walk). These results are close to those obtained by Hössinger et al. (2019) and Schmid et al. (2019) in their independently estimated models (both of which are based on the same dataset). Nonetheless, the joint estimation should be preferred. It is indeed superior over the independent estimation, as it permits the calculation of standard deviations for the VTAT, which is calculated from both types of choices. Also, it results in better model fit. Additionally, the joint estimation framework allows to better understand the user effect (according to Flügel (2014)) by a deeper decomposition into a VoL-related and VTAT-related parts. Although, the mode effect dominates in the VTTS differences, it might be partially reduced by means of segmentation according to age and trip purpose, which indicates that leisure trips reveal the lowest mode effect.

Moreover, for the first time we show the importance of the endogenous expenditure modelling, which has a considerable effect on VoL (decrease of 16.83%). Leaving out the expenditure equation would result in biased estimates of base line utility of work (θ_w), and total freely chosen expenses (Φ). We thus recommend using activity duration and expenditures in the model, both of which should be observed from the same individuals. In methodological terms, we have presented several innovations in this paper: We (i) estimated for the first time the full theoretical model of Jara-Díaz and Guevara (2003), (ii) extended the empirical framework of Munizaga et al. (2008) to incorporate the expenditure estimation, (iii) multiple trips per individual were allowed, and (iv) the panel structure of the underlying data is taken into account. The development of the procedure comes with its costs (as discussed below), but the solution is robust and runs on a conventional computer in reasonable time. Furthermore, it allows for a flexible definition of the number of equations (both continuous and discrete), varying number of alternatives in the choice sets, non-linearity of indirect utilities, inclusion of interaction terms, and usage of the produced likelihood function with other R packages.[9]

[9] We plan to present the finalized estimation procedure as an R package.

Some possible advances are left for future work. Instead of using the likelihood formulated in Munizaga et al. (2008), one could estimate the joint model in a full Bayesian framework. Also, the benefits of using probit instead of logit could be tested. Another problem is associated with the theoretical treatment of domestic work in the modelling framework; its classification as committed activity should be rethought. The nature of domestic (unpaid) work is arguably more similar to paid work than to eating or sleeping, in a sense that it can be outsourced to other persons. Although it is evident from our data that individuals responsible for more domestic chores work less and have a lower disposable income, the causality is up for discussion - work less because of more chores or more chores because of less work. Mostly, those engaging in more domestic chores are females, who work on average 9 h less per week in their official (paid) work and 8 h more in their unofficial domestic work than men (Hössinger et al., 2019). Mainly due to this, the value of leisure of females is worth 60% of males (5.86€/h vs 10.63€/h). It would be desirable to account for the monetary value of domestic work in one way or another. For the valuation, several possibilities exist: including the wage rate (Luxton, 1997, opportunity cost method), the market value of such domestic work (Folbre, 2006, market replacement cost method), the calculation of the monetary value of the goods/services produced (Luxton, 1997, input/output cost method) or the recent incorporation of domestic work in the time-use framework by Rosales-Salas and Jara-Díaz (2017). All options have their specifics but the consideration of unpaid work is likely to close the gender gap in the value of leisure.

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Appendix. Segmentation

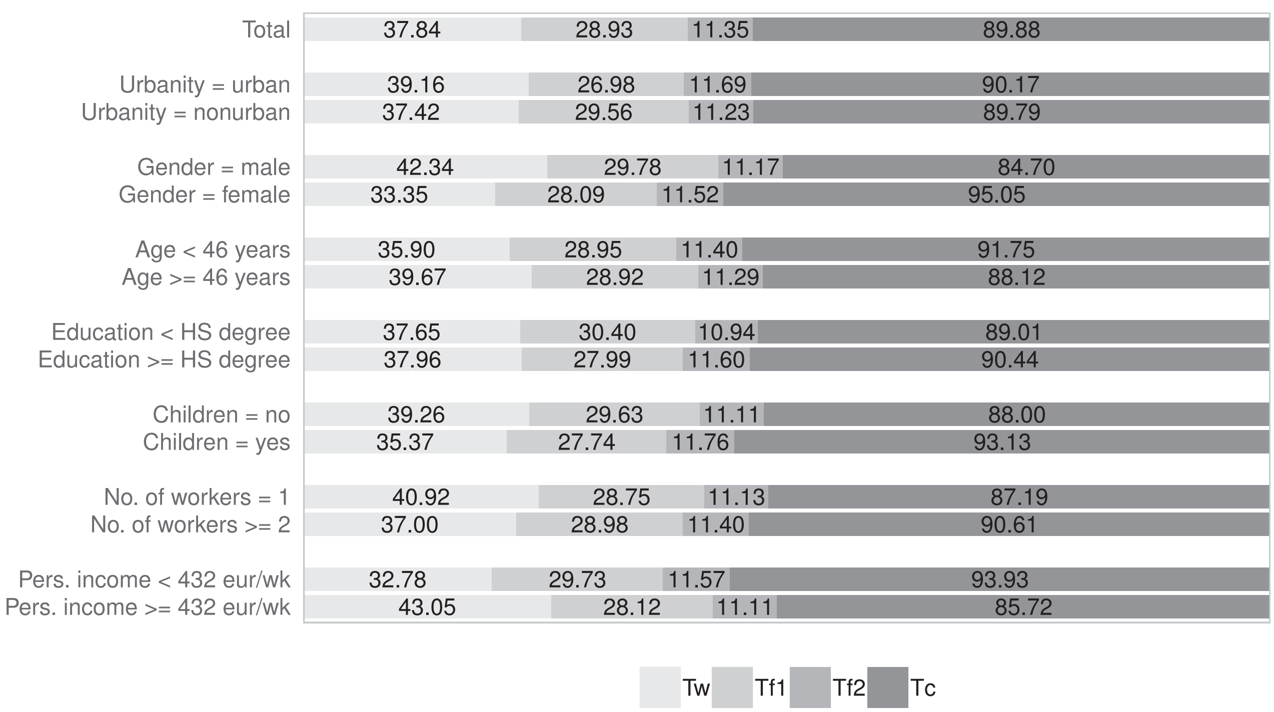

Fig. B.1. Distribution of activities in different segments, hours.

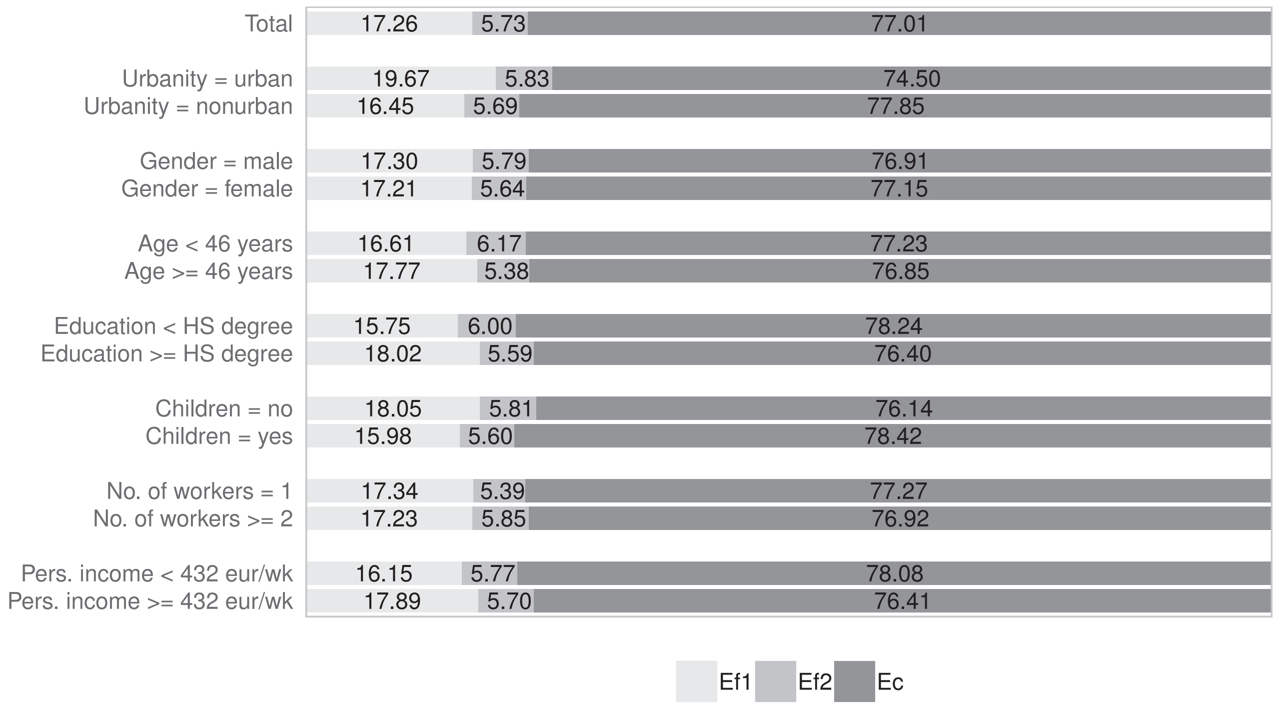

Fig. B.2. Distribution of expenditures in different segments, %.

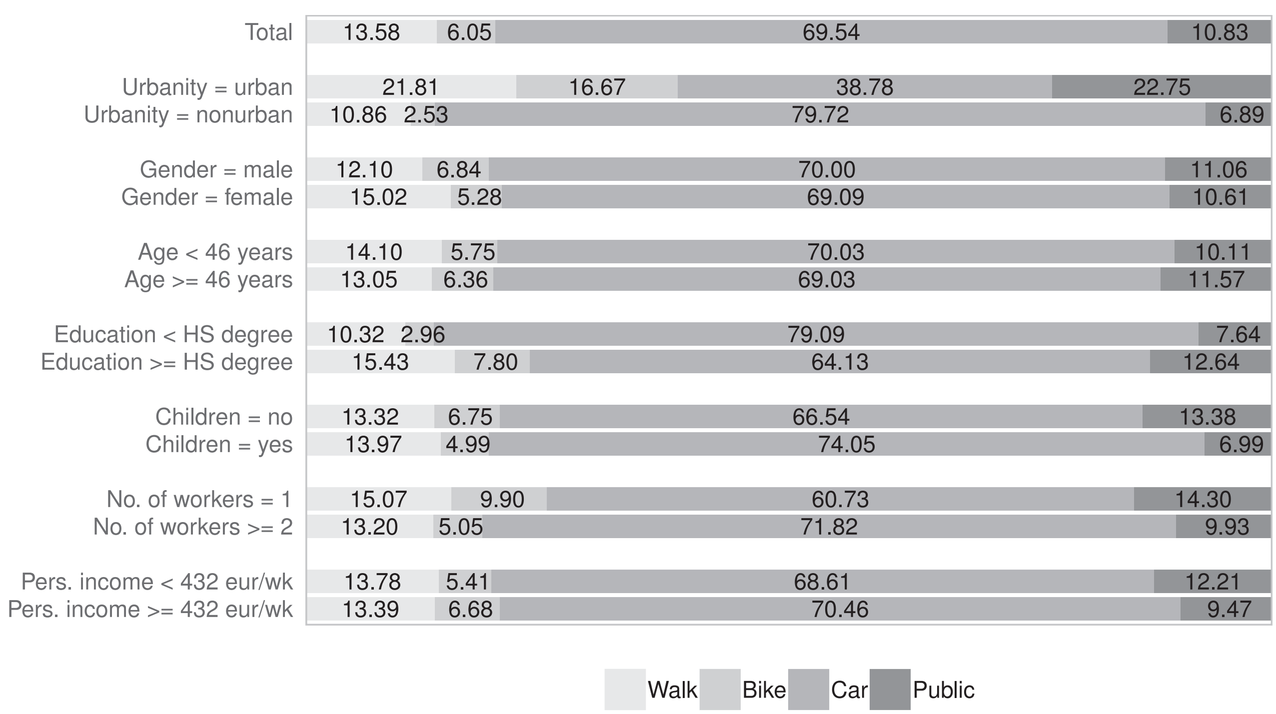

Fig. B.3. Distribution of transport modes in different segments, %.

Table B.1
Mean values and standard deviation (in brackets) of the model variables across different population segments (time-use variables: h/week; expenditures and wage: € /week).

Segmentation		Obs.	w	T_w	T_{f1}	T_c	E_{f1}	E_c	$T_w + T_c$	$wT_w - E_c$
Global		737	12.14	37.84	89.78	28.94	79.99	332.44	127.62	127.49
			[5.09]	[11.28]	[13.41]	[11.09]	[49.95]	[161.73]	[11.19]	[110.82]
Urbanity	= urban	178	12.53	39.16	90.07	26.98	95.44	343.18	129.23	147.35
			[5.00]	[11.97]	[12.60]	[11.64]	[54.86]	[148.01]	[11.79]	[114.76]
	= nonurban	559	12.01	37.42	89.69	29.56	75.07	329.02	127.11	121.16
			[5.12]	[11.04]	[13.66]	[10.84]	[47.29]	[165.84]	[10.95]	[108.88]
Gender	= male	368	12.76	42.34	84.60	29.78	93.53	394.74	126.94	148.06
			[5.79]	[9.28]	[11.64]	[11.58]	[53.94]	[175.37]	[11.56]	[123.33]
	= female	369	11.51	33.35	94.95	28.09	66.49	270.32	128.30	106.97
			[4.20]	[11.33]	[13.06]	[10.52]	[41.51]	[117.86]	[10.77]	[92.45]
Age	< 46 years	358	11.42	35.90	91.64	28.95	70.73	298.17	127.53	107.82
			[4.63]	[12.11]	[14.16]	[11.63]	[44.89]	[144.32]	[11.83]	[97.14]
	>= 46 years	379	12.82	39.67	88.03	28.92	88.74	364.82	127.70	146.06
			[5.41]	[10.12]	[12.42]	[10.57]	[52.89]	[170.57]	[10.56]	[119.53]
Education	< HS degree	288	10.35	37.65	88.90	30.40	62.87	285.99	126.55	102.23
			[3.39]	[10.68]	[13.40]	[10.56]	[34.00]	[120.83]	[11.00]	[80.06]
	>= HS degree	449	13.28	37.96	90.35	28.00	90.98	362.24	128.31	143.69
			[5.65]	[11.67]	[13.39]	[11.32]	[55.23]	[177.04]	[11.26]	[124.08]
Children	= no	467	11.80	39.26	87.92	29.63	81.80	324.69	127.19	142.65
			[5.04]	[10.18]	[12.33]	[11.06]	[49.04]	[157.57]	[11.05]	[109.98]
	= yes	270	12.72	35.37	92.99	27.74	76.87	345.85	128.37	101.26
			[5.15]	[12.62]	[14.56]	[11.05]	[51.45]	[168.14]	[11.40]	[107.51]
No. of workers	= 1	157	12.05	40.92	87.12	28.76	89.89	364.15	128.04	127.19
			[4.95]	[9.30]	[12.27]	[11.26]	[50.33]	[150.22]	[11.71]	[106.15]
	>= 2	580	12.16	37.00	90.50	28.98	77.31	323.86	127.50	127.57
			[5.14]	[11.63]	[13.62]	[11.05]	[49.56]	[163.78]	[11.05]	[112.14]
Pers. income	< 432 eur/wk	374	9.46	32.78	93.83	29.73	52.61	233.86	126.61	62.01
			[2.83]	[11.86]	[14.19]	[10.95]	[26.17]	[95.89]	[11.27]	[57.17]
	>= 432 eur/wk	363	14.89	43.05	85.61	28.12	108.21	434.02	128.66	194.95
			[5.43]	[7.75]	[11.11]	[11.19]	[52.87]	[152.70]	[11.01]	[112.29]

Table B.2
Trip purpose: Total. Indicators for different segments, value [s.d.].

Segmentation		VoL	VTAW	VTTS				VTAT			
				Walk	Bike	Car	Public	Walk	Bike	Car	Public
Global		9.29	−2.84	17.53	9.25	9.98	3.91	−8.23	0.05	−0.69	5.38
		[3.90]	[1.83]	[1.61]	[0.78]	[1.35]	[0.75]	[4.08]	[3.92]	[4.07]	[3.94]
Urbanity	= urban	6.76	−5.77	16.79	9.52	9.47	4.44	−10.04	−2.77	−2.71	2.32
		[2.61]	[3.10]	[0.31]	[0.18]	[0.17]	[0.48]	[2.63]	[2.62]	[2.62]	[2.66]
	= nonurban	7.61	−4.40	18.89	8.80	8.44	3.29	−11.28	−1.19	−0.83	4.32
		[3.33]	[2.55]	[1.51]	[1.04]	[1.76]	[1.01]	[3.66]	[3.49]	[3.78]	[3.48]
Gender	= male	10.63	−2.13	16.47	9.75	8.77	3.84	−5.84	0.88	1.86	6.79
		[4.71]	[1.32]	[0.28]	[0.37]	[0.56]	[0.42]	[4.72]	[4.72]	[4.75]	[4.73]
	= female	5.86	−5.64	21.55	8.56	9.86	4.39	−15.69	−2.70	−4.00	1.47
		[2.29]	[2.98]	[1.88]	[0.92]	[1.61]	[1.03]	[3.00]	[2.47]	[2.84]	[2.53]
Age	< 46 years	6.83	−4.58	17.20	7.34	5.49	2.29	−10.36	−0.50	1.34	4.54
		[2.83]	[2.73]	[0.74]	[0.47]	[0.62]	[0.49]	[2.94]	[2.88]	[2.91]	[2.88]
	>= 46 years	10.89	−1.93	13.78	8.33	9.51	3.64	−2.90	2.56	1.38	7.24
		[4.62]	[1.44]	[0.94]	[0.83]	[1.25]	[0.79]	[4.75]	[4.70]	[4.79]	[4.67]
Education	< HS degree	7.32	−3.04	20.70	9.56	10.72	4.89	−13.38	−2.24	−3.40	2.42
		[2.52]	[1.47]	[1.99]	[0.94]	[1.44]	[0.98]	[3.20]	[2.68]	[2.89]	[2.70]
	>= HS degree	8.05	−5.23	17.75	9.27	9.07	4.04	−9.71	−1.22	−1.02	4.01
		[3.44]	[3.04]	[0.56]	[0.50]	[0.49]	[0.40]	[3.49]	[3.48]	[3.47]	[3.46]
Children	= no	8.94	−2.86	15.39	9.21	8.76	4.42	−6.45	−0.28	0.18	4.52
		[3.80]	[1.64]	[0.69]	[0.57]	[1.09]	[0.70]	[3.88]	[3.85]	[3.99]	[3.88]
	= yes	7.37	−5.35	24.20	9.95	10.83	4.31	−16.83	−2.57	−3.46	3.06
		[3.07]	[3.19]	[1.19]	[0.86]	[0.73]	[0.84]	[3.30]	[3.20]	[3.15]	[3.16]
No. of workers	= 1	12.32	0.26	15.63	8.38	7.97	2.86	−3.31	3.94	4.34	9.46
		[5.08]	[0.35]	[1.18]	[0.88]	[1.18]	[0.69]	[5.21]	[5.16]	[5.22]	[5.14]
	>= 2	5.99	−6.17	16.14	8.61	8.16	3.64	−10.15	−2.62	−2.17	2.35
		[2.72]	[3.28]	[1.36]	[0.79]	[1.48]	[0.79]	[2.99]	[2.80]	[3.04]	[2.81]
Pers. income	< 432 eur/wk	4.37	−5.09	17.28	7.53	8.73	3.12	−12.91	−3.16	−4.36	1.25
		[1.64]	[2.48]	[1.45]	[0.83]	[1.44]	[0.97]	[2.18]	[1.83]	[2.17]	[1.90]
	>= 432 eur/wk	14.19	−0.70	18.47	10.37	10.24	5.27	−4.27	3.82	3.95	8.92
		[5.10]	[0.50]	[1.17]	[1.02]	[1.28]	[0.85]	[5.28]	[5.24]	[5.30]	[5.19]

Table B.3
Trip purpose: Work. Indicators for different segments, value [s.d.].

Segmentation		VoL	VTAW	VTTS				VTAT			
				Walk	Bike	Car	Public	Walk	Bike	Car	Public
Global		9.29	−2.84	26.55	8.79	9.96	3.77	−17.26	0.50	−0.66	5.52
		[3.90]	[1.83]	[2.48]	[1.06]	[2.36]	[1.46]	[4.48]	[4.01]	[4.55]	[4.18]
Urbanity	= urban	6.76	−5.77	21.20	7.15	4.11	1.20	−14.44	−0.39	2.65	5.56
		[2.61]	[3.10]	[0.37]	[0.28]	[0.10]	[0.52]	[2.64]	[2.63]	[2.62]	[2.67]
	= nonurban	7.61	−4.40	32.64	8.19	8.49	3.05	−25.03	−0.58	−0.88	4.55
		[3.33]	[2.55]	[2.94]	[1.19]	[2.58]	[1.57]	[4.45]	[3.54]	[4.25]	[3.70]
Gender	= male	10.63	−2.13	23.70	9.86	8.43	3.91	−13.07	0.77	2.20	6.72
		[4.71]	[1.32]	[0.32]	[0.46]	[0.70]	[0.68]	[4.72]	[4.74]	[4.77]	[4.76]
	= female	5.86	−5.64	34.04	7.90	11.22	3.79	−28.17	−2.03	−5.36	2.07
		[2.29]	[2.98]	[4.13]	[1.38]	[2.71]	[1.90]	[4.74]	[2.62]	[3.49]	[2.91]
Age	< 46 years	6.83	−4.58	28.90	6.41	5.01	2.12	−22.07	0.42	1.82	4.71
		[2.83]	[2.73]	[1.71]	[0.71]	[1.41]	[1.04]	[3.33]	[2.93]	[3.18]	[3.02]
	>= 46 years	10.89	−1.93	18.30	7.34	8.78	2.22	−7.41	3.55	2.11	8.66
		[4.62]	[1.44]	[1.36]	[1.03]	[1.74]	[1.15]	[4.86]	[4.74]	[4.92]	[4.74]
Education	< HS degree	7.32	−3.04	38.06	8.75	9.87	2.72	−30.74	−1.43	−2.56	4.60
		[2.52]	[1.47]	[3.42]	[1.34]	[2.25]	[1.40]	[4.24]	[2.83]	[3.34]	[2.86]
	>= HS degree	8.05	−5.23	24.67	8.21	6.92	2.75	−16.62	−0.17	1.13	5.30
		[3.44]	[3.04]	[0.99]	[0.59]	[0.85]	[0.77]	[3.58]	[3.50]	[3.55]	[3.53]
Children	= no	8.94	−2.86	22.92	7.87	7.12	2.88	−13.99	1.07	1.82	6.05
		[3.80]	[1.64]	[1.43]	[0.72]	[1.37]	[0.96]	[4.08]	[3.86]	[4.03]	[3.91]
	= yes	7.37	−5.35	35.45	10.82	14.47	6.14	−28.07	−3.45	−7.09	1.24
		[3.07]	[3.19]	[1.95]	[0.95]	[1.31]	[1.16]	[3.66]	[3.23]	[3.33]	[3.26]
No. of workers	= 1	12.32	0.26	24.50	7.42	8.92	3.53	−12.18	4.90	3.40	8.79
		[5.08]	[0.35]	[2.02]	[0.83]	[1.41]	[1.04]	[5.46]	[5.13]	[5.24]	[5.16]
	>= 2	5.99	−6.17	23.42	8.61	7.23	2.52	−17.43	−2.62	−1.24	3.47
		[2.72]	[3.28]	[2.10]	[0.94]	[2.06]	[1.22]	[3.37]	[2.86]	[3.35]	[2.94]
Pers. income	< 432 eur/wk	4.37	−5.09	28.36	6.78	8.02	2.14	−23.99	−2.41	−3.65	2.23
		[1.64]	[2.48]	[2.79]	[1.05]	[1.80]	[1.26]	[3.22]	[1.95]	[2.42]	[2.06]
	>= 432 eur/wk	14.19	−0.70	25.05	8.49	7.54	2.96	−10.86	5.70	6.65	11.23
		[5.10]	[0.50]	[2.76]	[1.23]	[2.28]	[1.61]	[5.85]	[5.27]	[5.62]	[5.37]

Table B.4
Trip purpose: Leisure. Indicators for different segments, value [s.d.].

Segmentation		VoL	VTAW	VTTS				VTAT			
				Walk	Bike	Car	Public	Walk	Bike	Car	Public
Global		9.29	−2.84	11.48	9.18	8.87	4.70	−2.19	0.11	0.43	4.59
		[3.90]	[1.83]	[1.61]	[1.28]	[1.85]	[1.43]	[4.08]	[4.01]	[4.22]	[4.06]
Urbanity	= urban	6.76	−5.77	12.80	11.11	14.07	7.04	−6.04	−4.35	−7.32	−0.28
		[2.61]	[3.10]	[0.40]	[0.30]	[0.33]	[0.66]	[2.64]	[2.63]	[2.63]	[2.70]
	= nonurban	7.61	−4.40	10.72	8.44	3.75	3.04	−3.11	−0.84	3.86	4.56
		[3.33]	[2.55]	[1.47]	[1.84]	[2.62]	[1.81]	[3.63]	[3.78]	[4.24]	[3.78]
Gender	= male	10.63	−2.13	10.41	8.95	5.97	3.16	0.22	1.68	4.66	7.47
		[4.71]	[1.32]	[0.68]	[0.51]	[1.07]	[0.61]	[4.77]	[4.73]	[4.84]	[4.75]
	= female	5.86	−5.64	14.31	9.00	5.47	5.27	−8.44	−3.13	0.39	0.59
		[2.29]	[2.98]	[1.85]	[1.88]	[3.02]	[2.37]	[2.99]	[2.98]	[3.82]	[3.32]
Age	< 46 years	6.83	−4.58	8.82	7.70	2.89	1.92	−1.99	−0.86	3.94	4.91
		[2.83]	[2.73]	[0.69]	[0.92]	[1.13]	[0.85]	[2.92]	[2.97]	[3.03]	[2.95]
	>= 46 years	10.89	−1.93	11.21	8.15	9.30	5.73	−0.33	2.74	1.58	5.16
		[4.62]	[1.44]	[1.33]	[1.48]	[2.47]	[1.85]	[4.80]	[4.84]	[5.18]	[4.89]
Education	< HS degree	7.32	−3.04	10.12	11.10	10.01	7.46	−2.80	−3.78	−2.69	−0.14
		[2.52]	[1.47]	[1.72]	[2.14]	[3.15]	[2.65]	[3.05]	[3.30]	[4.04]	[3.65]
	>= HS degree	8.05	−5.23	13.36	10.08	9.33	5.66	−5.31	−2.03	−1.28	2.39
		[3.44]	[3.04]	[0.86]	[1.13]	[0.86]	[0.67]	[3.56]	[3.64]	[3.55]	[3.51]
Children	= no	8.94	−2.86	10.10	10.05	9.58	7.01	−1.16	−1.12	−0.64	1.93
		[3.80]	[1.64]	[0.82]	[1.06]	[1.74]	[1.14]	[3.90]	[3.95]	[4.25]	[4.00]
	= yes	7.37	−5.35	15.37	8.36	2.53	2.18	−8.00	−0.98	4.84	5.19
		[3.07]	[3.19]	[0.99]	[1.10]	[1.81]	[1.60]	[3.20]	[3.27]	[3.52]	[3.42]
No. of workers	= 1	12.32	0.26	11.73	9.79	7.50	2.61	0.58	2.52	4.82	9.70
		[5.08]	[0.35]	[1.73]	[1.80]	[2.24]	[1.91]	[5.36]	[5.40]	[5.59]	[5.46]
	>= 2	5.99	−6.17	10.53	8.70	5.81	4.59	−4.54	−2.71	0.18	1.40
		[2.72]	[3.28]	[1.24]	[1.28]	[2.06]	[1.30]	[2.95]	[2.97]	[3.36]	[2.99]
Pers. income	< 432 eur/wk	4.37	−5.09	10.26	8.18	7.55	4.78	−5.89	−3.81	−3.18	−0.40
		[1.64]	[2.48]	[1.47]	[1.64]	[2.92]	[2.11]	[2.21]	[2.32]	[3.35]	[2.69]
	>= 432 eur/wk	14.19	−0.70	14.10	11.58	10.98	7.50	0.09	2.61	3.21	6.69
		[5.10]	[0.50]	[1.24]	[1.85]	[2.07]	[1.57]	[5.28]	[5.48]	[5.54]	[5.36]

Table B.5
Trip purpose: Other. Indicators for different segments, value [s.d.].

Segmentation		VoL	VTAW	VTTS				VTAT			
				Walk	Bike	Car	Public	Walk	Bike	Car	Public
Global		9.29	−2.84	14.55	9.76	11.11	3.25	−5.25	−0.47	−1.82	6.04
		[3.90]	[1.83]	[1.46]	[0.90]	[1.29]	[0.77]	[4.04]	[3.95]	[4.05]	[3.96]
Urbanity	= urban	6.76	−5.77	16.38	10.31	10.22	5.07	−9.62	−3.56	−3.46	1.68
		[2.61]	[3.10]	[0.38]	[0.21]	[0.29]	[0.38]	[2.64]	[2.62]	[2.63]	[2.64]
	= nonurban	7.61	−4.40	13.31	9.76	13.08	3.76	−5.70	−2.15	−5.48	3.84
		[3.33]	[2.55]	[1.16]	[1.20]	[1.85]	[1.07]	[3.53]	[3.55]	[3.81]	[3.50]
Gender	= male	10.63	−2.13	15.30	10.45	11.90	4.45	−4.67	0.18	−1.27	6.18
		[4.71]	[1.32]	[0.45]	[0.51]	[0.33]	[0.30]	[4.73]	[4.73]	[4.72]	[4.72]
	= female	5.86	−5.64	16.32	8.79	12.89	4.11	−10.46	−2.93	−7.03	1.75
		[2.29]	[2.98]	[1.43]	[1.13]	[2.23]	[1.38]	[2.72]	[2.59]	[3.32]	[2.75]
Age	< 46 years	6.83	−4.58	13.87	7.90	8.58	2.81	−7.04	−1.06	−1.75	4.02
		[2.83]	[2.73]	[0.75]	[0.63]	[1.21]	[0.78]	[2.95]	[2.92]	[3.12]	[2.96]
	>= 46 years	10.89	−1.93	11.84	9.51	10.44	2.97	−0.95	1.38	0.45	7.91
		[4.62]	[1.44]	[0.94]	[0.92]	[1.41]	[0.91]	[4.77]	[4.74]	[4.91]	[4.77]
Education	< HS degree	7.32	−3.04	13.92	8.82	12.28	4.50	−6.60	−1.50	−4.96	2.81
		[2.52]	[1.47]	[1.58]	[1.10]	[1.77]	[1.12]	[2.97]	[2.75]	[3.09]	[2.76]
	>= HS degree	8.05	−5.23	15.23	9.51	10.95	3.70	−7.18	−1.46	−2.90	4.35
		[3.44]	[3.04]	[0.55]	[0.47]	[1.09]	[0.77]	[3.48]	[3.47]	[3.59]	[3.51]
Children	= no	8.94	−2.86	13.14	9.72	9.57	3.37	−4.21	−0.78	−0.64	5.57
		[3.80]	[1.64]	[0.59]	[0.68]	[1.12]	[0.80]	[3.85]	[3.87]	[4.00]	[3.90]
	= yes	7.37	−5.35	21.78	10.67	15.49	4.63	−14.40	−3.29	−8.11	2.75
		[3.07]	[3.19]	[1.67]	[1.21]	[1.82]	[0.81]	[3.52]	[3.33]	[3.60]	[3.18]
No. of workers	= 1	12.32	0.26	10.65	7.93	7.51	2.43	1.67	4.39	4.81	9.89
		[5.08]	[0.35]	[0.85]	[0.84]	[1.45]	[0.91]	[5.14]	[5.15]	[5.29]	[5.17]
	>= 2	5.99	−6.17	14.48	8.51	11.43	3.82	−8.48	−2.52	−5.44	2.18
		[2.72]	[3.28]	[1.34]	[0.85]	[1.33]	[0.84]	[2.99]	[2.83]	[3.00]	[2.83]
Pers. income	< 432 eur/wk	4.37	−5.09	13.23	7.63	10.61	2.44	−8.85	−3.25	−6.24	1.93
		[1.64]	[2.48]	[1.27]	[0.98]	[1.59]	[1.00]	[2.06]	[1.90]	[2.27]	[1.91]
	>= 432 eur/wk	14.19	−0.70	16.25	11.04	12.21	5.36	−2.05	3.16	1.98	8.84
		[5.10]	[0.50]	[1.10]	[1.00]	[1.50]	[0.97]	[5.25]	[5.23]	[5.36]	[5.21]

Table B.6
Estimation results for different segments, estimate [s.d.].

Segmentation		Φ	ϕ_1	θ_1	θ_w	β_{cost}	β_{walk}	β_{bike}	β_{car}	β_{PT}
Global		0.30	0.17	0.74	−0.26	−0.59	−0.17	−0.09	−0.10	−0.04
		[0.02]	[0.01]	[0.01]	[0.05]	[0.06]	[0.01]	[0.01]	[0.01]	[0.01]
Urbanity	= urban	0.51	0.34	0.73	−0.80	−0.60	−0.17	−0.10	−0.10	−0.04
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	= nonurban	0.37	0.21	0.74	−0.49	−0.55	−0.17	−0.08	−0.08	−0.03
		[0.02]	[0.01]	[0.01]	[0.04]	[0.03]	[0.01]	[0.01]	[0.02]	[0.01]
Gender	= male	0.31	0.19	0.74	−0.20	−0.55	−0.15	−0.09	−0.08	−0.04
		[0.01]	[0.00]	[0.00]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]
	= female	0.42	0.24	0.74	−0.73	−0.55	−0.20	−0.08	−0.09	−0.04
		[0.03]	[0.02]	[0.01]	[0.09]	[0.04]	[0.01]	[0.01]	[0.02]	[0.01]
Age	< 46 years	0.38	0.22	0.75	−0.55	−0.68	−0.20	−0.08	−0.06	−0.03
		[0.01]	[0.01]	[0.01]	[0.03]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]
	>= 46 years	0.27	0.16	0.74	−0.16	−0.65	−0.15	−0.09	−0.10	−0.04
		[0.02]	[0.02]	[0.01]	[0.08]	[0.04]	[0.01]	[0.01]	[0.01]	[0.01]
Education	< HS degree	0.32	0.19	0.76	−0.35	−0.69	−0.24	−0.11	−0.12	−0.06
		[0.02]	[0.01]	[0.01]	[0.02]	[0.06]	[0.01]	[0.01]	[0.01]	[0.01]
	>= HS degree	0.39	0.24	0.72	−0.56	−0.54	−0.16	−0.08	−0.08	−0.04
		[0.01]	[0.01]	[0.01]	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]
Children	= no	0.34	0.19	0.75	−0.28	−0.55	−0.14	−0.08	−0.08	−0.04
		[0.01]	[0.01]	[0.01]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
	= yes	0.32	0.22	0.72	−0.58	−0.49	−0.20	−0.08	−0.09	−0.04
		[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]
No. of workers	= 1	0.22	0.15	0.76	0.02	−0.70	−0.18	−0.10	−0.09	−0.03
		[0.02]	[0.01]	[0.01]	[0.02]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]
	>= 2	0.46	0.27	0.74	−0.87	−0.62	−0.17	−0.09	−0.08	−0.04
		[0.02]	[0.01]	[0.01]	[0.05]	[0.05]	[0.01]	[0.01]	[0.01]	[0.01]

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Table B.6 (*continued*)

Segmentation		Φ	ϕ_1	θ_1	θ_w	β_{cost}	β_{walk}	β_{bike}	β_{car}	β_{PT}
Pers. income	< 432 eur/wk	0.38	0.26	0.75	−0.87	−0.66	−0.19	−0.08	−0.10	−0.03
		[0.02]	[0.01]	[0.01]	[0.05]	[0.03]	[0.01]	[0.01]	[0.02]	[0.01]
	>= 432 eur/wk	0.31	0.16	0.73	−0.05	−0.54	−0.17	−0.09	−0.09	−0.05
		[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]

Segmentation		$\gamma_{L,\ walk}$	$\gamma_{L,bike}$	$\gamma_{L,car}$	$\gamma_{L,PT}$	$\gamma_{W,\ walk}$	$\gamma_{W,bike}$	$\gamma_{W,car}$	$\gamma_{W,PT}$
Global		0.06	0.00	0.01	−0.01	−0.09	0.00	0.00	0.00
		[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.01]
Urbanity	= urban	0.04	−0.02	−0.05	−0.03	−0.04	0.02	0.05	0.03
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	= nonurban	0.07	0.00	0.04	0.00	−0.12	0.01	0.00	0.00
		[0.01]	[0.01]	[0.02]	[0.01]	[0.01]	[0.01]	[0.02]	[0.01]
Gender	= male	0.06	0.01	0.03	0.01	−0.07	0.00	0.00	0.00
		[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	= female	0.07	0.00	0.04	−0.01	−0.11	0.01	−0.01	0.01
		[0.02]	[0.01]	[0.02]	[0.02]	[0.02]	[0.01]	[0.02]	[0.02]
Age	< 46 years	0.10	0.00	0.03	0.00	−0.13	0.01	0.01	0.00
		[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
	>= 46 years	0.03	0.00	0.00	−0.02	−0.05	0.01	0.01	0.02
		[0.01]	[0.01]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Education	< HS degree	0.12	−0.02	0.01	−0.03	−0.20	0.01	0.01	0.02
		[0.01]	[0.02]	[0.03]	[0.02]	[0.01]	[0.01]	[0.02]	[0.02]
	>= HS degree	0.04	−0.01	0.00	−0.01	−0.06	0.01	0.02	0.01
		[0.00]	[0.01]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.01]
Children	= no	0.05	−0.01	−0.01	−0.02	−0.07	0.01	0.01	0.01
		[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.01]	[0.01]
	= yes	0.07	0.01	0.07	0.02	−0.09	−0.01	−0.03	−0.02
		[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.01]	[0.00]
No. of workers	= 1	0.05	−0.02	0.01	0.00	−0.10	0.01	−0.01	−0.01
		[0.01]	[0.01]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]
	>= 2	0.06	0.00	0.02	−0.01	−0.07	0.00	0.01	0.01
		[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Pers. income	< 432 eur/wk	0.08	−0.01	0.01	−0.02	−0.12	0.01	0.01	0.01
		[0.01]	[0.01]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]
	>= 432 eur/wk	0.04	−0.01	−0.01	−0.02	−0.06	0.02	0.02	0.02
		[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.01]	[0.02]	[0.01]

Table B.7
Estimation results for different segments, estimate [s.d.].

Segmentation		α_{bike}	α_{PT}	α_{car}	β_{t2bus}	$\beta_{servInt}$	β_{stops}	β_{HhPark}	$\beta_{JobPark}$	β_{MgPark}	$\rho_{Tw\&Tf1}$
Global		−3.20	−2.19	−1.98	−0.06	−0.03	−0.42	0.59	0.63	−1.20	−0.70
		[0.03]	[0.10]	[0.03]	[0.01]	[0.01]	[0.06]	[0.06]	[0.03]	[0.10]	[0.02]
Urbanity	= urban	−2.28	−1.83	−1.95	−0.06	−0.02	−0.37	0.28	0.41	−0.97	−0.72
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.03]
	= nonurban	−3.85	−2.38	−1.64	−0.05	−0.03	0.08	0.07	0.69	−1.03	−0.67
		[0.02]	[0.04]	[0.07]	[0.02]	[0.01]	[0.08]	[0.06]	[0.04]	[0.09]	[0.02]
Gender	= male	−2.73	−2.05	−2.19	−0.06	−0.03	−0.27	0.58	0.65	−0.59	−0.70
		[0.00]	[0.00]	[0.01]	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
	= female	−3.94	−1.83	−1.99	−0.08	−0.04	−0.17	0.38	0.46	−1.17	−0.70
		[0.09]	[0.08]	[0.07]	[0.01]	[0.01]	[0.06]	[0.09]	[0.06]	[0.05]	[0.02]
Age	< 46 years	−3.38	−2.41	−2.49	−0.04	−0.03	−0.23	0.58	0.53	−1.19	−0.71
		[0.03]	[0.04]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.02]	[0.02]
	>= 46 years	−3.03	−1.82	−1.66	−0.07	−0.03	−0.40	0.43	0.70	−0.86	−0.68
		[0.03]	[0.05]	[0.04]	[0.01]	[0.01]	[0.04]	[0.06]	[0.05]	[0.08]	[0.02]
Education	< HS degree	−4.02	−2.66	−2.80	−0.06	−0.04	−0.01	0.98	0.70	−0.87	−0.71
		[0.01]	[0.05]	[0.03]	[0.01]	[0.01]	[0.06]	[0.03]	[0.04]	[0.03]	[0.02]
	>= HS degree	−2.82	−1.90	−2.05	−0.07	−0.03	−0.21	0.35	0.50	−1.09	−0.67
		[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]
Children	= no	−2.69	−1.57	−1.57	−0.06	−0.03	−0.14	0.28	0.64	−0.91	−0.71
		[0.02]	[0.03]	[0.03]	[0.01]	[0.00]	[0.02]	[0.02]	[0.03]	[0.01]	[0.02]
	= yes	−3.73	−2.76	−2.67	−0.07	−0.02	−0.29	0.67	0.53	−1.11	−0.66
		[0.02]	[0.01]	[0.02]	[0.02]	[0.01]	[0.04]	[0.01]	[0.01]	[0.02]	[0.03]

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Table B.7 (*continued*)

Segmentation		α_{bike}	α_{PT}	α_{car}	β_{t2bus}	$\beta_{servInt}$	β_{stops}	β_{HhPark}	$\beta_{JobPark}$	β_{MgPark}	$\rho_{Tw\&Tf1}$
No. of workers	= 1	−2.79	−1.97	−1.69	−0.06	−0.03	−0.24	−0.10	0.51	−1.06	−0.75
		[0.03]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.03]	[0.04]	[0.03]
	>= 2	−3.15	−1.96	−2.25	−0.07	−0.03	−0.27	0.61	0.54	−0.97	−0.67
		[0.02]	[0.02]	[0.02]	[0.01]	[0.00]	[0.05]	[0.03]	[0.03]	[0.03]	[0.02]
Pers. income	< 432 eur/wk	−3.70	−2.20	−2.30	−0.06	−0.03	−0.24	0.71	0.63	−1.15	−0.72
		[0.11]	[0.04]	[0.04]	[0.02]	[0.01]	[0.05]	[0.05]	[0.03]	[0.06]	[0.02]
	>= 432 eur/wk	−2.97	−2.05	−1.94	−0.07	−0.03	−0.26	0.25	0.47	−0.62	−0.66
		[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.03]	[0.03]	[0.03]	[0.02]
Segmentation		$\rho_{Tw\&Ef1}$	$\rho_{Tw\&walk}$	$\rho_{Tw\&bike}$	$\rho_{Tw\&PT}$	$\rho_{Tw\&car}$	$\rho_{Tf1\&Ef1}$	$\rho_{Tf1\&walk}$	$\rho_{Tf1\&bike}$	$\rho_{Tf1\&PT}$	$\rho_{Tf1\&car}$
Global		0.40	−0.07	−0.10	0.03	−0.18	−0.44	0.22	0.21	0.15	0.28
		[0.03]	[0.02]	[0.03]	[0.03]	[0.04]	[0.03]	[0.02]	[0.03]	[0.03]	[0.03]
Urbanity	= urban	0.38	−0.02	−0.08	0.18	−0.17	−0.42	0.17	0.23	0.02	0.25
		[0.05]	[0.04]	[0.04]	[0.05]	[0.05]	[0.05]	[0.04]	[0.04]	[0.05]	[0.05]
	= nonurban	0.30	0.00	−0.01	0.06	0.03	−0.38	0.20	0.08	0.15	0.18
		[0.04]	[0.03]	[0.04]	[0.04]	[0.05]	[0.03]	[0.03]	[0.04]	[0.04]	[0.04]
Gender	= male	0.30	−0.01	−0.05	0.18	0.05	−0.37	0.10	0.07	−0.03	−0.04
		[0.04]	[0.04]	[0.04]	[0.04]	[0.05]	[0.04]	[0.03]	[0.04]	[0.05]	[0.05]
	= female	0.46	−0.04	−0.08	0.04	−0.24	−0.48	0.25	0.25	0.17	0.38
		[0.03]	[0.03]	[0.04]	[0.04]	[0.05]	[0.03]	[0.03]	[0.04]	[0.04]	[0.04]
Age	< 46 years	0.39	−0.02	0.01	0.09	−0.08	−0.39	0.19	0.15	0.20	0.27
		[0.03]	[0.03]	[0.04]	[0.04]	[0.05]	[0.04]	[0.03]	[0.04]	[0.04]	[0.05]
	>= 46 years	0.38	−0.12	−0.21	−0.02	−0.26	−0.44	0.27	0.26	0.09	0.28
		[0.04]	[0.03]	[0.04]	[0.05]	[0.05]	[0.04]	[0.03]	[0.04]	[0.04]	[0.05]
Education	< HS degree	0.19	−0.04	−0.08	0.06	0.00	−0.29	0.28	0.18	0.08	0.25
		[0.06]	[0.04]	[0.06]	[0.07]	[0.07]	[0.05]	[0.04]	[0.06]	[0.07]	[0.06]
	>= HS degree	0.45	−0.06	−0.12	0.04	−0.20	−0.47	0.15	0.16	0.10	0.20
		[0.03]	[0.03]	[0.03]	[0.03]	[0.04]	[0.03]	[0.03]	[0.03]	[0.04]	[0.04]
Children	= no	0.38	−0.12	−0.05	−0.01	−0.22	−0.41	0.29	0.20	0.20	0.32
		[0.04]	[0.03]	[0.04]	[0.04]	[0.05]	[0.04]	[0.03]	[0.04]	[0.04]	[0.04]
	= yes	0.40	−0.02	−0.14	0.17	−0.11	−0.43	0.07	0.15	0.02	0.12
		[0.04]	[0.05]	[0.05]	[0.05]	[0.06]	[0.05]	[0.04]	[0.05]	[0.05]	[0.06]
No. of workers	= 1	0.38	−0.18	0.04	0.19	−0.05	−0.36	0.35	0.16	0.12	0.30
		[0.05]	[0.05]	[0.06]	[0.06]	[0.08]	[0.05]	[0.05]	[0.05]	[0.06]	[0.06]
	>= 2	0.37	−0.04	−0.14	0.02	−0.21	−0.42	0.19	0.22	0.14	0.27
		[0.03]	[0.03]	[0.03]	[0.03]	[0.04]	[0.03]	[0.02]	[0.03]	[0.03]	[0.03]
Pers. income	< 432 eur/wk	0.27	0.08	0.14	0.20	0.06	−0.31	0.17	0.09	0.06	0.24
		[0.04]	[0.03]	[0.04]	[0.04]	[0.05]	[0.04]	[0.03]	[0.04]	[0.04]	[0.05]
	>= 432 eur/wk	0.46	−0.10	−0.27	−0.05	−0.21	−0.48	0.15	0.15	0.11	0.09
		[0.04]	[0.03]	[0.04]	[0.05]	[0.04]	[0.04]	[0.03]	[0.04]	[0.05]	[0.04]

Table B.8
Estimation results for different segments, estimate [s.d.].

Segmentation		$\rho_{Ef1\&walk}$	$\rho_{Ef1\&bike}$	$\rho_{Ef1\&PT}$	$\rho_{Ef1\&car}$	f_{bike}	f_{PT}	f_{car}	ω_{walk}	ω_{bike}	ω_{PT}	ω_{car}
Global		−0.33	−0.41	−0.47	−0.58	−0.92	−0.41	1.20	2.68	4.19	1.89	2.26
		[0.02]	[0.02]	[0.02]	[0.02]	[0.03]	[0.03]	[0.07]	[0.03]	[0.03]	[0.02]	[0.03]
Urbanity	= urban	−0.11	−0.31	−0.26	−0.22	−0.85	−0.48	1.01	2.22	3.81	1.85	2.77
		[0.03]	[0.04]	[0.04]	[0.05]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	= nonurban	−0.34	−0.26	−0.41	−0.57	−0.94	−0.38	1.14	3.48	4.90	1.87	2.01
		[0.03]	[0.04]	[0.04]	[0.03]	[0.06]	[0.06]	[0.08]	[0.07]	[0.06]	[0.03]	[0.13]
Gender	= male	−0.24	−0.34	−0.42	−0.40	−0.87	−0.45	1.04	2.73	4.13	1.86	2.33
		[0.03]	[0.03]	[0.04]	[0.03]	[0.00]	[0.01]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]
	= female	−0.34	−0.41	−0.44	−0.65	−0.84	−0.25	1.06	2.92	4.41	1.67	2.14
		[0.03]	[0.03]	[0.03]	[0.03]	[0.06]	[0.07]	[0.06]	[0.07]	[0.05]	[0.06]	[0.08]
Age	< 46 years	−0.33	−0.39	−0.46	−0.52	−0.87	−0.42	0.99	2.75	3.98	2.04	2.41
		[0.03]	[0.03]	[0.03]	[0.03]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]
	>= 46 years	−0.37	−0.40	−0.47	−0.64	−0.82	−0.36	1.23	2.97	4.60	1.72	1.94
		[0.03]	[0.03]	[0.03]	[0.02]	[0.03]	[0.02]	[0.04]	[0.03]	[0.04]	[0.02]	[0.06]
Education	< HS degree	−0.30	−0.12	−0.53	−0.47	−0.83	−0.50	1.05	2.97	6.02	2.28	2.50
		[0.04]	[0.06]	[0.06]	[0.05]	[0.02]	[0.02]	[0.06]	[0.04]	[0.03]	[0.09]	[0.03]
	>= HS degree	−0.32	−0.43	−0.42	−0.58	−0.87	−0.48	1.04	2.61	3.85	1.65	2.18
		[0.02]	[0.03]	[0.03]	[0.02]	[0.01]	[0.01]	[0.02]	[0.03]	[0.01]	[0.01]	[0.01]

(*continued on next page*)

Table B.8 (*continued*)

Segmentation		$\rho_{Ef1\&walk}$	$\rho_{Ef1\&bike}$	$\rho_{Ef1\&PT}$	$\rho_{Ef1\&car}$	f_{bike}	f_{PT}	f_{car}	ω_{walk}	ω_{bike}	ω_{PT}	ω_{car}
Children	= no	−0.34	−0.37	−0.43	−0.58	−0.84	−0.45	1.07	2.72	4.58	1.64	2.28
		[0.03]	[0.03]	[0.04]	[0.03]	[0.02]	[0.04]	[0.02]	[0.03]	[0.01]	[0.02]	[0.03]
	= yes	−0.40	−0.40	−0.42	−0.63	−0.85	−0.50	1.00	2.64	3.85	2.40	2.27
		[0.03]	[0.04]	[0.04]	[0.03]	[0.01]	[0.01]	[0.01]	[0.02]	[0.01]	[0.02]	[0.01]
No. of workers	= 1	−0.44	−0.40	−0.39	−0.40	−0.89	−0.50	0.97	2.18	4.16	1.34	2.45
		[0.04]	[0.05]	[0.06]	[0.06]	[0.01]	[0.01]	[0.02]	[0.02]	[0.05]	[0.01]	[0.05]
	>= 2	−0.34	−0.40	−0.49	−0.66	−0.80	−0.34	1.06	2.95	4.28	1.84	2.12
		[0.02]	[0.03]	[0.03]	[0.02]	[0.06]	[0.07]	[0.02]	[0.05]	[0.02]	[0.02]	[0.02]
Pers. income	< 432 eur/wk	−0.30	−0.37	−0.44	−0.56	−0.94	−0.52	0.97	2.99	4.65	2.00	2.15
		[0.03]	[0.04]	[0.04]	[0.04]	[0.05]	[0.06]	[0.03]	[0.05]	[0.11]	[0.04]	[0.06]
	>= 432 eur/wk	−0.30	−0.41	−0.46	−0.50	−0.86	−0.49	1.07	2.84	4.23	1.69	2.23
		[0.03]	[0.03]	[0.03]	[0.02]	[0.02]	[0.01]	[0.02]	[0.03]	[0.02]	[0.02]	[0.01]
Segmentation		$\alpha_{L,bike}$	$\alpha_{L,PT}$	$\alpha_{L,car}$	σ_{Tw}	σ_{Tf1}	σ_{Ef1}	$\alpha_{W,bike}$	$\alpha_{W,PT}$	$\alpha_{W,car}$		
Global		0.43	0.64	0.47	61.50	64.70	36.50	−1.05	−0.97	−1.19		
		[0.03]	[0.04]	[0.03]	[0.36]	[0.19]	[0.63]	[0.05]	[0.02]	[0.05]		
Urbanity	= urban	0.55	0.58	1.10	63.20	65.40	39.70	−0.89	−0.82	−1.50		
		[0.00]	[0.00]	[0.00]	[0.33]	[0.46]	[0.73]	[0.00]	[0.00]	[0.00]		
	= nonurban	0.69	1.07	0.22	59.50	63.20	35.00	−1.55	−1.29	−1.49		
		[0.07]	[0.06]	[0.04]	[0.48]	[0.31]	[0.39]	[0.03]	[0.07]	[0.06]		
Gender	= male	0.64	0.58	0.60	60.00	70.30	41.00	−0.95	−0.82	−1.30		
		[0.01]	[0.01]	[0.00]	[0.63]	[0.30]	[0.47]	[0.00]	[0.01]	[0.00]		
	= female	0.40	0.79	−0.05	62.80	59.20	30.00	−1.10	−1.23	−1.09		
		[0.04]	[0.10]	[0.06]	[0.53]	[0.29]	[0.68]	[0.10]	[0.07]	[0.07]		
Age	< 46 years	0.84	0.88	0.78	63.10	67.80	32.40	−1.45	−1.25	−1.80		
		[0.03]	[0.03]	[0.01]	[0.61]	[0.57]	[0.59]	[0.02]	[0.01]	[0.05]		
	>= 46 years	0.21	0.74	0.17	58.10	62.00	37.60	−0.99	−1.01	−0.82		
		[0.04]	[0.03]	[0.04]	[0.64]	[0.19]	[0.69]	[0.04]	[0.05]	[0.04]		
Education	< HS degree	1.32	1.40	0.74	61.60	62.20	29.00	−1.96	−1.98	−1.90		
		[0.02]	[0.05]	[0.05]	[0.38]	[0.49]	[0.60]	[0.03]	[0.03]	[0.05]		
	>= HS degree	0.40	0.49	0.43	59.90	67.30	38.30	−0.84	−0.87	−1.26		
		[0.01]	[0.01]	[0.02]	[0.61]	[0.19]	[0.44]	[0.02]	[0.01]	[0.02]		
Children	= no	0.62	0.75	0.58	61.40	63.70	34.60	−1.07	−0.87	−1.16		
		[0.06]	[0.02]	[0.01]	[0.32]	[0.35]	[0.49]	[0.02]	[0.03]	[0.02]		
	= yes	0.34	0.85	0.12	55.40	63.50	37.30	−0.84	−1.07	−1.10		
		[0.01]	[0.01]	[0.02]	[0.35]	[0.28]	[0.90]	[0.01]	[0.01]	[0.01]		
No. of workers	= 1	0.22	0.10	0.11	60.90	64.10	36.50	−1.04	−1.01	−1.00		
		[0.02]	[0.02]	[0.04]	[0.63]	[0.44]	[1.28]	[0.01]	[0.03]	[0.04]		
	>= 2	0.60	0.77	0.33	60.80	63.90	33.20	−0.78	−1.01	−1.25		
		[0.02]	[0.04]	[0.04]	[0.47]	[0.16]	[0.37]	[0.06]	[0.03]	[0.03]		
Pers. income	< 432 eur/wk	0.72	0.89	0.53	64.20	63.80	23.40	−1.34	−1.27	−1.58		
		[0.07]	[0.06]	[0.07]	[0.43]	[0.34]	[0.59]	[0.06]	[0.03]	[0.04]		
	>= 432 eur/wk	0.34	0.57	0.30	47.30	64.70	43.50	−0.86	−0.88	−1.02		
		[0.01]	[0.01]	[0.02]	[0.57]	[0.34]	[0.64]	[0.01]	[0.02]	[0.02]		

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Assessing the value of household work based on wages demanded on online platforms for substitutes

Simona Jokubauskaitė[1] · Alyssa Schneebaum[2]

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Abstract
We propose an improved method to assess the economic value of unpaid housework and childcare. Existing literature has typically assigned a minimum, generalist or specialist's wage, or the performer's opportunity cost to the hourly value of these activities. Then it was used to calculate macro-level value based on the number of hours spent in this work. In this paper, instead of imputing an average or minimum wage for housework and childcare to determine a value to the work, we use the actual local wage rate requested for these services from providers on online platforms. Applying this method to Austrian Time Use Survey data shows that the value of unpaid childcare and housework, had it been paid, would be equivalent to about 22% of the 2018 GDP.

Keywords Unpaid work · Valuation · Domestic work · Housework · Care work

JEL Classification C81 · J22 · J13 · G59

1 Introduction

No economy would function without adequate care for young children (who will grow to become economic agents) or the housework necessary to sustain people. Since care- and housework in one's own home is largely unpaid, it is unclear how to measure the economic value of this work. To understand the economics of the household, though, it is important to understand better the economic value of the work produced within it.

✉ Alyssa Schneebaum
alyssa.schneebaum@wu.ac.at

[1] Institute of Statistics, University of Natural Resources and Life Sciences, Peter-Jordan-Straße 82/I, 1190 Vienna, Austria

[2] Department of Economics, Vienna University of Economics and Business, Welthandelsplatz 1, 1020 Vienna, Austria

The literature quantifying the worth of house- and care work[1] assigns a value to the number of hours spent in this work (most commonly calculated using time-use surveys) in one of three main ways. On the one hand, there are two "input" methods: the opportunity cost approach and the market replacement cost approach. On the other hand, there is an "output" method. The first of these three – the opportunity cost approach – assumes that time spent on unpaid work is at the expense of earning a market wage. Many calculations of opportunity costs simply use an average wage rate of all employed people (Ahmad & Koh, 2011); others estimate a potential wage rate even for people outside the paid labor force (Gammage, 2010; Schmid et al., 1999). The second approach – the market replacement approach – imputes wages that reflect the market price of the respective tasks, using either the average wage rate of a general housekeeper (Ahmad & Koh, 2011; Varjonen et al., 2014) or including multiple wage rates of specialists in matched occupations (Hamdad, 2003; Landefeld et al., 2005). Sometimes, calculations with minimum wages for these tasks are included to provide lower-bound estimates (Landefeld et al., 2005). Some studies using the market replacement approach also account for the intensity of care (e.g. physical and development care) and incorporate supervisory care (Suh & Folbre, 2016; Mullan, 2010). Finally, the output approach quantifies the value of the output of unpaid work, measuring the service price of, for example, a kilogram of washing or ironing (Holloway et al., 2002), or a child taken care of (Mullan, 2010; Yoon, 2014). In practice, many studies in the literature calculate a value of unpaid work using more than just one of these methods, providing instead a battery of potential wage rates in order to give a range of estimates for the value of domestic and care work. The fact that many studies report values calculated with several different approaches speaks to the lack of any "best" practice in this literature.

In considering how to better and consistently quantify the value of unpaid house- and care work, we propose a new approach to introduce more precision into wage estimates in the specialist replacement cost method. In particular, we use the wages demanded for housework and childcare on actual online platforms to get the market price of the work performed, disaggregated by region at the NUTS-2 level. We then apply these values to region-specific time-use statistics to compute the aggregate value of the work.

The use of online platforms to organize work has become increasingly prevalent (see, for example, Katz and Krueger (2019) and Kässi and Lehdonvirta (2018)), not least to match households with babysitters and cleaners. Our approach thus allows the literature to keep up with the changing nature of the organization and payment for this work. National accounting offices, who are already working to supplement GDP measures with satellite accounts (European Communities, 2003), can find our approach particularly useful.

In relation to other literature on this topic, we consider our approach to fit into the specialist framework, in that we assume that it is specialists who offer childcare and cleaning services on online platforms. Table 1 shows that there are varying degrees of "specialization" among the workers on the platforms we use; those with more years of experience also demand higher wages.

[1] We use the terms "housework" and "domestic" work interchangeably, and always differentiate these activities from childcare. Childcare and housework/domestic work are defined in footnote 3 below.

Table 1 Distribution of advertisements by years of experience[a]

	Domestic		Childcare	
Experience	Mean	N	Mean	N
0–4	11.18	6555	10.58	8975
5–9	11.57	3377	10.94	3625
10+	11.96	4518	11.66	2070

[a]Not all workers used in the main analysis below indicate their years of experience

The benefits of the calculation method described here are five-fold. First, our approach does not rely on hypothetical considerations about the value of the worker's time to measure an opportunity cost of doing the work, estimates of which differ widely based on education, professional experience, and socioeconomic background (Schmid et al., 1999). Instead, our approach takes the value of the work to be the market value that the worker herself demands. Second, our approach uses relatively local wage rates (NUTS-2 level), meaning that the application of our method would produce more accurate local estimates of the value of household and care work. Third, the approach avoids applying a minimum wage to a job that may actually receive higher remuneration, making it more accurate. Fourth, the wage data are reported per hour, unlike in labor force surveys, where hourly wage data are often imprecisely computed because they are calculated using other variables such as yearly income, hours worked per week, and weeks worked per year. Finally, our approach uses free and real-time data on wages paid, instead of relying on this information from labor force surveys. The latter are costly in terms of time as well as money and their publication typically has a long delay after data collection. Researchers can use our method in any country that collects time-use data and in which there are platforms organizing these services.

In the next section, we show an application of our method with the wage data collected from online platforms. We estimate the aggregated value of typically unpaid work by applying wage rates to time-use data from the same region.

2 Application

We use data from Austria for our empirical application. Data on time-use come from the 2008–2009[2] time-use survey (TUS) conducted by Statistik Austria (2020). According to the TUS, the following tasks are considered housework: cooking, cleaning, laundry, gardening, repairs, shopping for the household, household management, travel related to housework, and other housework activities. Childcare comprises primary care of the child, helping with the child with schoolwork, recreation with the child, accompanying the child to appointments, travel related to

[2] There are more recent time-use data for Austria available from the Mobility-Activity-Expenditure Diary (MAED) survey conducted in 2015, but these data comprise only 748 individuals and do not report childcare as a separate activity. As reported in Hössinger et al. (2019), time-use patterns did not change significantly between the national time use survey in 2008/2009 and the MAED survey in 2015, and there is no reason to suspect that it would have changed significantly in the time to 2019.

Table 2 Time spent in childcare and housework (minutes/day), 15–64 year olds

		Primary activity only			Primary and secondary		
Segment (N)	Activity	Total	Women	Men	Total	Women	Men
Total (6215)	Childcare	34.0	46.8	18.7	43.1	60.1	22.7
	Housework	172.6	224.6	110.3	182.0	238.2	114.5
w/ children (1542)	Childcare	117.0	156.3	66.1	148.0	200.4	80.0
	Housework	175.7	239.3	93.3	188.3	257.7	98.5
w/o children (4673)	Childcare	6.6	9.0	3.9	8.5	11.7	4.8
	Housework	171.6	219.5	115.6	179.9	231.4	119.5

childcare, and other childcare activities. These activities likely make up a large portion of all activities related to domestic and childcare work. However, any other kind of work in these areas is not captured in our wage calculations - which implies that our estimates are a lower-bound value for domestic and childcare work. Table 2 shows the average number of minutes spent per day on childcare and household work for 15–64 year olds. It is calculated based on whether the activity was primary or secondary (the latter are activities that are done simultaneously with the primary activity, such as listening to music while primarily cooking). Individuals in the survey spent on average about 34 min in childcare and 172 min in household work per day. Disaggregating these figures into households with and without children under 10 reveals that–not surprisingly–those with children spent on average almost 2 h more on childcare. Parents spend "only" 2 h per day on childcare because the Austrian Time Use Survey does not include information on supervisory responsibilities that encompasses activities like being present while a child naps. The survey thus cannot capture the element of childcare that relates to the worth of what care work *prevents* a caregiver from doing, such as working for pay outside of the home.

To calculate the monetary value of this work, we use two prominent online platforms to obtain the wages for housework and childcare: www.haushaltshilfe24.at (household help 24) and www.babysitter24.at, respectively. These sites are intended for private, individual users (both suppliers and demanders of housework and childcare services), but some agencies may covertly find work for their employees via these platforms.

In June 2019 we collected data on the wages demanded from workers offering childcare and housework[3] services. The listings on these websites give an hourly wage, which we use in our calculations. Some of the arrangements are for one-time jobs, while others may be requesting repeated work, such as weekly babysitting. We take the hourly wage as listed, regardless of the regularity with which the work was meant to be completed. In addition, the data from the listings do not allow us to differentiate between work to be completed during regular business hours versus other times of the

[3] Cooking is a prominent activity in the time-use data, which may be too particular to include in the general housework category. To test this, we used the fact that some workers mention their willingness to cook in their personal descriptions on the websites. We divided our sample into advertisements with and without cooking and performed a t-test for independent samples. The results show that there is no statistically significant difference in wages between these two groups. Thus, the mean wage used in the analysis was constructed from all available activities without further differentiating cooking.

Table 3 Number of advertisements by NUTS-2 region in Austria

	Domestic		Childcare	
	Women	Men	Women	Men
All	12,871	1584	14,041	753
Vienna	5003	633	5796	425
Lower Austria	1881	233	2046	100
Styria	1781	207	1583	65
Upper Austria	1728	221	1525	61
Carinthia	639	82	695	21
Tyrol	595	69	983	32
Salzburg	573	75	679	28
Burgenland	423	39	331	8
Vorarlberg	248	25	403	13

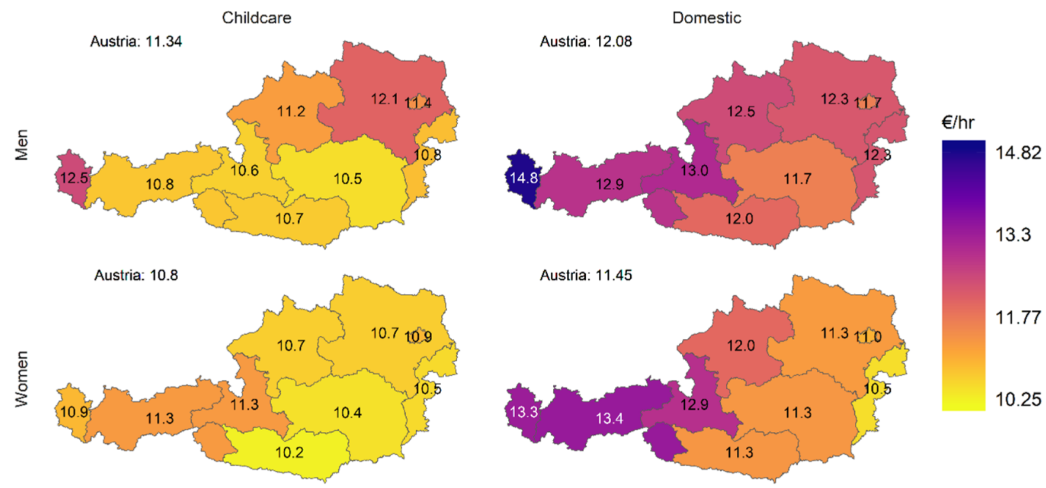

Fig. 1 Average wage rates of domestic work, EUR/h

day, like overnight care shifts, though the latter could be paid differently than daytime work. We cannot specify whether these wages are the worker's gross wages or net of taxes, because tax payments depend on total income. Workers with very low income do not pay any income taxes; some of the workers offering services on this platform may fall into this group, while others will have to pay taxes on their income.

We filtered listings according to postal area and worker gender. It would have been interesting to further observe wages for immigrant versus native workers, but the data do not contain information on country of origin or family migration background. We took the wages demanded from around 15,000 observations from each online platform. Almost 90% of advertisements offering services come from women (Table 3). We calculate the average wage demanded for household work and childcare at the NUTS-2 level, for each of the nine regions of Austria (Fig. 1).

Figure 1 shows the average wage rate demanded for household work and for babysitting services, broken down by gender and region. There is significant variation in the hourly wage of these services across regions, but overall, an hour of childcare costs about €11 and household work costs closer to €12. Interestingly, the

Table 4 Average value of domestic work as percentage of GDP

Activities included	Women	Men	Total
Primary activity only			
Total	14.21	7.41	21.62
Childcare	2.23	0.94	3.17
Housework	11.98	6.47	18.45
Both, weighted[a]			
Total	9.06	4.62	13.68
Childcare	1.58	0.64	2.22
Housework	7.48	3.98	11.46
Primary and secondary			
Total	15.54	7.84	23.39
Childcare	2.83	1.13	3.96
Housework	12.72	6.71	19.42

[a]Time spent on the activity was weighted by 0.6 if it was a primary activity and by 0.4 if it was a secondary activity, as suggested in Apps and Rees (2011, FN 24). These weights were only used in the row "both, weighted" in the ablet; the data in the other rows were unweighted

average hourly wage demanded by men on these platforms is higher than the wage demanded by women.

3 A macro-level estimate for the value of unpaid work

We next apply the region-specific average wage rates to the average time spent on household and childcare activities in each region. The goal of this exercise is to determine the value of the work as a percentage of GDP, which is the standard measure of the value of unpaid work in the literature. We use the most recent GDP data at the time of research available for Austria, namely data from 2018, which was equal to about €386 billion. There is a 1-year mismatch in the time dimensions of our wage and GDP data; the platform wage data were collected in 2019 and the most recent GDP figures are from 2018. To correct this mismatch, we deflate the wage rates using the national consumer price index,[4] Table 4 presents results of our final percentage estimates.

When considering primary activities only, we find that the value of housework and childcare amounts to about 22% of the 2018 GDP; when using both primary and secondary activities, the value is near 23.4%. Ahmed and Koh (2011) have also calculated the value of labor for domestic and care work in Austria compared to the total value of the 2008 GDP using the same time-use data, but with different valuation methods and wage data (in particular, OECD PPP wage data) and including several activities not used in our study (in particular, care for adult household

[4] We use the country-level consumer price index to deflate the wage rate. It would have been beneficial to deflate prices at the regional level but region-specific consumer price indices are unavailable for Austria.

members, care for non-household members, and volunteer work). Using average generalist wages for primary activities, they estimate that labor costs for housework and care work was equivalent to 24% of GDP in 2008. When using instead an opportunity cost approach, the estimate is 41% of GDP in 2008. Schappelwein (2018) has applied the generalist approach to the same Austrian time-use data but with data on net wages from European Union Survey on Income and Living Conditions. She computed a labor value for housework and care of dependent people (children and others) equivalent to 27% of GDP in 2008. These comparisons – which all use the same time-use data – show that the valuation of domestic work is sensitive to the methodological approach and, crucially, the wage data employed. We argue that our approach is superior to previous methods, primarily because it uses what is likely the closest true value of the market cost of an hour of this work. In addition, with more widespread use of the internet, more services will be offered online and thus the estimates of wage rates using our method will improve.

4 Discussion

The main contribution of this paper has been to introduce a new approach to the valuation of domestic work. By using easily accessible and free information on actual wages, our method helps form a better understanding of the market value of household work and childcare. Calculating this worth sheds light on the economic value of such work. We find that this work would be additionally worth about 22% of GDP if it were paid at going market prices; this estimate is close to the value created by the industry sector in Austria. The approach proposed in this paper can be particularly useful to national accounting offices while developing household-specific satellite accounts to supplement measures of the gross domestic product.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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The role of unpaid domestic work in explaining the gender gap in the (monetary) value of leisure

Simona Jokubauskaitė[1C)], Reinhard Hössinger[2)], Sergio Jara-Díaz[3)], Stefanie Peer[4)], Alyssa Schneebaum[5)], Basil Schmid[6)], Florian Aschauer[2)], Regine Gerike[7], Kay W. Axhausen[6)], Friedrich Leisch[1)]

[C)] For correspondence and/or offprints; e-mail: simona.jokubauskaite@boku.ac.at
[1)] BOKU, Institute of Statistics,
Austria
simona.jokubauskaite@boku.ac.at, Friedrich.Leisch@boku.ac.at
[2)] BOKU, Institute for Transport Studies,
Austria
r.hoessinger@boku.ac.at, florian.aschauer@boku.ac.at
[3)] Universidad de Chile, Department of Civil Engineering,
Instituto Sistemas Complejos de Ingeniería (ISCI),
Chile jaradiaz@ing.uchile.cl
[4)] WU, Institute for Multi-Level Governance and Development,
Austria
stefanie.peer@wu.ac.at
[5)] WU, Department of Economics,
Austria
Alyssa.Schneebaum@wu.ac.at
[6)] ETH Zurich, Institute for Transport Planning and Systems (IVT),
Switzerland
basil.schmid@ivt.baug.ethz.ch, axhausen@ivt.baug.ethz.ch Germany
regine.gerike@tu-dresden.de

Abstract The value of travel time savings (VTTS) representing the willingness to pay to reduce travel time, consists of two components: the value of liberating time (equal to the value of leisure (VoL)) and the value of time assigned to travel (VTAT), representing the travel conditions of a trip. Their relative values indicate which dimension to emphasize when investing in transport: speed or comfort. In this paper, we formulate and estimate a framework aimed at the improvement in the estimation of the VoL. By introducing a novel treatment of time assigned to domestic work, we consider that unpaid labor should be assigned a wage rate as a measure of the expenses avoided when assigning time to those chores. We use state-of-the-art data on time use and expenses as well as online data on gig workers collected in Austria, and apply the time-use and expenditure model of Jara-Diaz et al. (2008). The wage rates for paid and unpaid work were combined to re-formulate the budget constraint, which affected women more than men due to the higher involvement of the former in domestic activities. Compared against the original estimation, the VoL changed from €10/hr for men and €6/hr for women to €9/hr for both genders, which in turn yields a larger average VTAT, which becomes positive for public transport. As a conclusion, the novel treatment of domestic labor contributes to closing the gap in the VoL between genders and highlights the power of unveiling the components behind the VTTS. The empirical findings imply that investments in travel time reductions rather than in comfort should be prioritized, given the very good conditions of public transport in Austria.

Keywords domestic work; gender budget analysis; value of leisure; gender pay gap; value of time assigned to travel; value of travel time savings

Contents

1 Introduction

The value of travel time savings (VTTS) has long been considered a key input in the appraisal of transport projects. The usual interpretation has been that the larger the VTTS the more important is the reduction in trip time. After all, it represents the willingness to pay to diminish travel time by one unit and is usually estimated with some form of travel choice model. However, behind this single figure there are two elements that contribute to the size of its value: the valuation that the individual assigns to the liberated time - the value of leisure (VoL)- and the valuation that the individual assigns to the conditions of travel – the value of time assigned to travel (VTAT). For short, the VTTS increases not only with the value of time assigned to alternative (rewarding) activities but also with the unpleasantness of the trip, components that were first identified and estimated by Jara-Díaz and Guevara (2003). Identifying the VoL and VTAT "illuminates the emphasis that decision makers should put when deciding on investments in the transport sector whenever speed and comfort collide" (Jara-Diaz, 2020). It is therefore vital to empirically determine which of the two components dominates, which can be achieved by estimating the VoL from time use models[1].

For more than 50 years, it has been widely agreed that time is a scarce resource and that it has value (Becker, 1965; Johnson, 1966; Oort, 1969; DeSerpa, 1971). The main two techniques for its valuation are the contingent valuation method (CVM) and time-use models (TUM). Both methods offer a way of assigning value to various types of activities such as work, travel and leisure. Only TUM, however, rely on the observed assignment of time to activities based on the assumption that behind that assignment underlies an implicit subjective valuation of the time spent on those activities, which can be revealed by modeling behavior. Since the formulation of this microeconomic model by Jara-Díaz and Guevara (2003), a series of improvements have been made in the estimation of the VoL from TUM. Nevertheless, this framework still lacks an adequate treatment of some activities: "In order to estimate more reliable VoL, there are two activity types that should be explored and studied with great care: what can be called maintenance activities (sleep, eat), and domestic work." Jara-Diaz (2020).

Domestic work (cleaning, cooking, childcare) presents various particular characteristics. First, it consumes a significant proportion of time - as shown in Table 1 -, certainly comparable to paid work time, and sometimes larger[2]. Second, in most (if not in all) countries the majority of domestic work is done by women, such that even if time assigned to paid work is lower for women, total work time is in most countries larger for them, as shown in Figure 1. Third, domestic work is an activity that can be delegated to a third (hired) party (Reid, 1934); it is not a tertiary activity (like transport) as defined by Burda et al. (2007). Several recent studies have reported gender specific differences in the VoL (e.g. Konduri et al., 2011; Verbooy et al., 2018; Hössinger et al., 2019; Jokubauskaitė et al., 2019) which we believe to be connected with an inadequate treatment of domestic work. This is what we want to examine in this paper, giving domestic work a new treatment while employing the TUM framework. As explained above, this is necessary for the estimation of a more reliable VoL and, ultimately, of better estimates of the VTAT. After all, undergoing domestic work means "[...] money saved that is traded for time assigned; the connection with the VoL is quite evident"(Jara-Diaz, 2020). This paper hence focuses on the role of unpaid domestic work in explaining the gap between gender specific (monetary) values of leisure (VoL) and proposes our approach as a possible advancement in estimating the components of the VTTS.

[1] Recent estimations of VoL and VTAT in Austria (Hössinger et al., 2019; Jokubauskaitė et al., 2019) and Swiss (Schmid et al., 2020) show that VTAT can indeed dominate for public transport.

[2] Australia, Poland, Italy, Spain, Belgium, Greece.

Table 1 Distribution of time in different countries, %

Country	Unpaid work	Paid work or study	Leisure	Personal care	Other
Mexico	18.74	24.90	12.34	42.53	1.49
Australia	16.89	16.54	19.53	45.66	1.39
Slovenia	16.02	18.38	21.57	43.62	0.42
Poland	15.70	15.15	19.29	46.33	3.53
Portugal	15.45	20.49	16.71	46.92	0.42
Italy	15.21	12.27	22.45	49.15	0.92
Spain	15.05	14.03	21.95	48.04	0.93
Denmark	15.04	15.61	22.84	45.79	0.72
Ireland	14.76	18.79	21.90	42.76	1.79
Belgium	14.64	13.25	25.61	46.15	0.35
Estonia	14.42	17.75	21.49	46.02	0.35
New Zealand	14.17	18.75	20.90	45.00	1.18
Austria	14.03	21.28	20.22	43.89	0.58
Hungary	13.90	19.25	19.32	47.39	0.14
Turkey	13.80	13.87	19.65	48.29	4.39
United States	13.71	20.10	19.67	44.88	1.63
Finland	13.67	15.91	22.99	44.48	2.95
Sweden	13.64	19.04	21.32	42.41	3.60
Latvia	13.62	22.91	18.67	44.53	0.27
Germany	13.61	17.21	23.00	45.00	1.18
Norway	13.61	16.75	25.57	43.56	0.50
United Kingdom	13.51	18.21	21.22	44.81	2.25
Greece	13.40	12.63	24.50	48.99	0.49
Netherlands	13.39	19.49	20.53	44.28	2.32
India	13.27	20.48	17.64	47.74	0.86
Canada	12.92	21.16	19.37	44.21	2.36
South Africa	12.66	16.71	21.24	48.27	1.12
France	12.58	14.18	20.38	52.24	0.62
China (People's Republic of)	11.38	23.59	15.82	48.16	1.04
Korea	9.43	24.18	20.24	45.26	0.89
Japan	9.17	25.18	19.33	43.06	3.27

Source: OECD (2018)

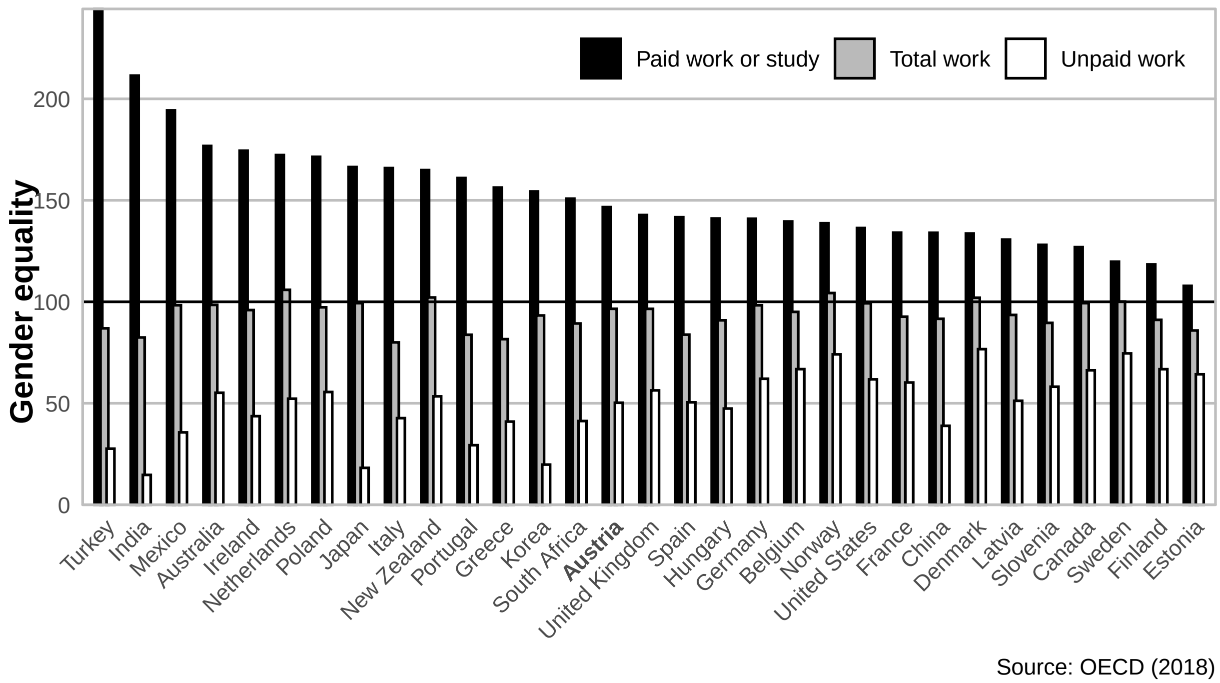

Fig. 1 Average daily work-time by men as proportion of work-time by women. Latest year available data reported

In time use models, it is common to treat only the official (paid) work as "work" and domestic (unpaid) work as a committed activity[3]. This assumption is questionable, since work is work and time is money. So far, only some studies have factored in the domestic work while estimating the value of leisure. In a recent paper by Verbooy et al. (2018), the value of unpaid work in the Netherlands was estimated with CVM (€16/hr) and was shown to be close to the wage rate of housekeepers (€14/hr, Hakkaart-van Roijen et al. (2015)). However, the CVM approach is based on stated (declared) preferences and "stated behavior does not necessarily align with the actual behavior of respondents" (Verbooy et al., 2018). The TUM framework from Jara-Díaz et al. (2008) was recently extended in a related direction as well: Rosales-Salas and Jara-Díaz (2017) introduced external providers (employed personnel). The estimation of such model is, however, not always possible or efficient. Domestic chores are mostly performed by household members and not by employed personnel. Therefore, obtaining a sufficiently large sample size of households that employ personnel is likely to be difficult in most countries.

The original Jara-Díaz and Guevara (2003) framework offers a convenient way for the VoL estimation and has been used in many studies (Munizaga et al., 2008; Jara-Díaz et al., 2008; Jara-Díaz and Astroza, 2013; Jara-Díaz et al., 2013; Konduri et al., 2011; Jara-Díaz et al., 2016). The most recent and advanced applications are Hössinger et al. (2019) and Jokubauskaitė et al. (2019), who work with the same innovative data gathered in Austria as this paper. Although, the used dataset is very informative (Aschauer et al., 2018), no information on external providers or stated preferences for time-use allocation was gathered. Thus, neither an estimation similar to the one included in Rosales-Salas and Jara-Díaz (2017) nor the application of CVM is possible.

In both recent investigations of the VTTS based on the same dataset as this paper (Hössinger et al., 2019; Jokubauskaitė et al., 2019), the VoL[4] has been estimated for different demographic and socioeconomic segments. The papers separately but consistently report that the estimated VoL of women is only half of

[3]Committed activities are activities that are necessary for personal/household maintenance and on which individuals do not want to spend more time than needed.

[4]The VoL corresponds to the resource value of time in DeSerpa (1971).

the VoL of men[5]. Moreover, the largest recorded gender-specific differences in time allocation are between time spent in domestic work (men: 10.11hr/week; women: 18.42hr/week) and paid work (men: 42.34hr/week; women: 33.35hr/week). This phenomenon is not unique for Austria, and has been well documented around the world. According to Alonso et al. (2019), women are typically involved in two hours more of unpaid work per day than men. Also, having a partner and/or children hampers their leisure time (Alonso et al., 2019; Wallace and Young, 2010). Argyrous and Rahman (2017) have observed that regardless of their earnings and working hours, women spend more time and take on more responsibility for childcare (Heiland et al., 2017). Thus, women are involved in a "second shift" and they suffer from the resulting time poverty (Hochschild and Machung, 2012; Schor, 2008; Hochschild, 1997; Vickery, 1977). The reduction of labor market hours caused by parenthood or taking care of a dependent person (Chaykowski and Powell, 1999; Bloemen et al., 2010; European Commission, 2017) reduces their financial security as well.

The microeconomic theory of labor supply introduced by Becker (1965) assumes that individuals decide on the amount of time devoted to work by maximizing their utility functions subject to time and income constraints. According to Becker's time-use model (TUM), income is defined by the time assigned to work and the hourly wage rate and the VoL is set to equal the wage rate. Thus, the gender-specific VoL gap is the difference between the corresponding wage rates. Based on Becker's framework, Johnson (1966), DeSerpa (1971) and Jara-Díaz and Guevara (2003) provided subsequent theoretical work on the valuation of leisure and other types of time. Another evolutionary extension of TUM came with the previously mentioned introduction of external providers by Rosales-Salas and Jara-Díaz (2017). They show that the exclusion of external providers, results in an overestimation of the VoL. Yet, if nobody is hired, they treat the same work as a committed activity, pushing it to the shadows of the economic domain, as it does not generate any income. This is well reflected in the words of Margaret G. Reid written in the beginning of the 20th century: "the more we have concentrated on money values the more we have overlooked that part of our economic system which is not organized on a profit basis" (Reid, 1934).

The majority of existing studies examining the economic valuation of unpaid work focuses on the inclusion of unpaid work into the calculation of the gross domestic product (GDP). Depending on methodological and geographical differences, the value of unpaid work ranges from 10 % up to 60% of GDP. For instance, Bridgman et al. (2018) estimate the average value of unpaid work to be 35 % of GDP for their sample of 43 countries (covering nearly a half of the world population). Payne and Vassilev (2018) have reported a 60% unpaid work value share for the UK, Schwarz and Schwahn (2016) 40% for Germany, and Schmid et al. (2002) 40-60% for Switzerland. Such a sizable contribution is certainly relevant and prompts a more careful look at the role it plays in the calculation of the values of work and leisure for men and women in the TUM framework.

In this study, we investigate the effect of the valuation (i.e. monetarization) of domestic work on gender-specific differences in the values of leisure (VoL) derived from the time-use and expenditure model (Jara-Díaz et al., 2008). This in turn will improve the VTAT estimation and will allow including a more balanced gender analysis (BGA) into the CBA of transport related projects. We estimate the TUM from Jara-Díaz et al. (2008) using a novel and very rich dataset collected in Austria in 2015, which is based on a Mobility-Activity-Expenditure Diary (MAED) described in Aschauer et al. (2018). Unlike the original framework, we consider that individuals who undertake domestic work could have hired a third person. We assume that they could have paid a market wage rate, which individuals choose not to pay avoiding expenses. In our study, different forms for the calculation of the wage rate associated with unpaid work have been analyzed in depth, with the result that we impute it using online data on the market price of various domestic work services in Austria.

The modeling framework (a system of non-linear equations) is introduced in the next section. Afterwards, in Section 3 we describe the data, offer new ways to consider a wage rate for domestic work, estimate the model using the re-calculated wage rate including a sensitivity analysis, and present the modeling results. These are then compared to the ones obtained with the original data in Section 4, where we show a) that the gap in the VoL between genders almost disappears if the value of domestic work is considered, and b) that the global VoL increases. This suggests an updated method to estimate time values for the analysis of transport related

[5] Konduri et al. (2011) also estimated a higher VoL for men (€39.94/hr) than for women (€24.56/hr) (for more segmentation comparisons see Hössinger et al. (2019) Table 1).

projects.

2 Theoretical Model

The frameworks of Becker (1965), DeSerpa (1971) or Heckman (1974) laid the foundation of the time-use incorporation into the consumer theory. Each advancement resulted in the derivation of new indicators - value of leisure ("resource value of time", (DeSerpa, 1971)), value of travel time savings (Bates, 1987; Jara-Díaz, 1998), and value of time assigned to travel (Jara-Díaz et al., 2008). In this article, we use the same theoretical model as Hössinger et al. (2019) and Jokubauskaitė et al. (2019), which was first presented in Jara-Díaz and Guevara (2003) and further advanced by Jara-Díaz et al. (2008). The Cobb-Douglas utility function maximized by each individual is given as:

$$U = \Omega T_w^{\theta_w} \prod_{i=1}^{n} T_i^{\theta_i} \prod_{j=1}^{m} E_j^{\phi_j} \tag{1}$$

In Eq. (1), the utility U is a function of the time assigned to work (T_w), the time assigned to activity i (T_i), and the expenditure assigned to good j (E_j). Ω is a utility constant, the exponents $\{\theta_w, \theta_i, \phi_j\}$ represent the elasticities of utility with respect to work, activities, and expenditures on goods. By dropping out Ω and taking a monotonous transformation of U, the maximization problem can be rewritten as:

$$\max ln(U) = \max \left(\theta_w ln(T_w) + \sum_{i=1}^{n} \theta_i ln(T_i) + \sum_{j=1}^{m} \phi_j ln(E_j) \right) \tag{2}$$

Subject to:

$$\tau - T_w - \sum_{i=1}^{n} T_i = 0 \ (\mu) \qquad \text{(time constraint)} \tag{3}$$

$$wT_w + I - \sum_{j=1}^{m} E_j \geq 0 \ (\lambda) \qquad \text{(budget constraint)} \tag{4}$$

$$T_i - T_i^{Min} \geq 0 \ (\kappa_i) \qquad \text{(technical constraints on activities)} \tag{5}$$

$$E_j - E_j^{Min} \geq 0 \ (\eta_j) \qquad \text{(technical constraints on goods)} \tag{6}$$

Here w is the wage rate; I is the income not related to work; τ is total available time in the work-leisure cycle (for example 168 hours - per week). There are m expenditure groups with index set G and n activity categories with index set A. Also, goods and activities are separated into freely chosen (index set G_f and A_f) and committed (index set G_c and A_c). The latter goods and activities restrict their freely chosen counterparts and are deduced from the observations ($T_g = T_g^{Min}, g \in A_c$ and $E_k = E_k^{Min}, k \in G_c$). The sum of such activities and goods is introduced in the time and budget constraints as T_c and E_c, respectively (Hössinger et al., 2019). To solve this maximization problem, we assume that individuals freely choose how much to work. After application of the Lagrange method with multipliers λ and μ, one gets an expression of optimal T_w, E_i and T_j (for more details on the derivation see Hössinger et al., 2019):

$$T_w^* = \frac{\left((\Phi+\theta_w)(\tau-T_c)+(\Theta+\theta_w)\frac{E_c}{w}\right)\pm\sqrt{D}}{2(\Theta+\Phi+\theta_w)} \tag{7}$$

$$\text{with } D = \left((\Phi+\theta_w)(\tau-T_c)+(\Theta+\theta_w)\frac{E_c}{w}\right)^2 - 4(\Theta+\Phi+\theta_w)\theta_w(\tau-T_c)\frac{E_c}{w}$$

$$T_{fi}^* = \frac{\theta_i}{\Theta}(\tau - T_w^* - T_c) \tag{8}$$

$$E_{fj}^* = \frac{\phi_j}{\Phi}(wT_w^* - E_c) \tag{9}$$

Here $\Theta = \sum_{i\in A_f}\theta_i$ and $\Phi = \sum_{j\in G_f}\phi_j$. $T_c = \sum_{i\in A_c} T_i^{min}$ and $E_c = (\sum_{j\in G_c} E_j^{min} - I)$ correspond to total committed time and expenditures, respectively.

The model given by the nonlinear equation system (Eq. 7 - 9) can be estimated using the maximum likelihood (ML) method. After the normalization of the full model (by setting $\Theta = 1$) there are at least 4 parameters left to estimate $\{\theta_w, \Phi, \theta_1, \phi_1\}$ and 2 parameters to calculate ($\theta_2 = 1 - \theta_1$ and $\phi_2 = \Phi - \phi_1$). In the original model of Jara-Díaz and Guevara (2003), a different normalization was used; the choice of normalization does not affect the estimates, as there is a direct relationship between $\{\alpha, \beta\}$ used in Jara-Díaz and Guevara (2003) and $\{\theta_w, \Theta, \Phi\}$ used here (based on Jara-Díaz et al. (2008)). In Jara-Díaz et al. (2008) (Eq. 24 and 25) it was shown that the VoL and value of time assigned to work (VTAW) can be calculated using the estimates of Φ and θ_w:

$$VoL = \frac{\partial U \backslash \partial T_i}{\partial U \backslash \partial E_j} = \frac{\mu}{\lambda} = \frac{\Theta(wT_w - E_c)}{\Phi(\tau - T_w - T_c)} \overbrace{=}^{\Theta=1} \frac{(wT_w - E_c)}{\Phi(\tau - T_w - T_c)} \tag{10}$$

$$VTAW = \frac{\partial U \backslash \partial T_w}{\lambda} = \frac{\theta_w}{\Phi}\frac{wT_w - E_c}{T_w} = VoL - w \tag{11}$$

Here λ and μ are marginal utilities of income and time, respectively. Thus, the VoL is calculated as a ratio between two marginal utilities. This result was derived from the First Oder Conditions (FOC) of the maximization problem for Eq. (2) and constraints of time (Eq. 3) and budget (Eq. 4). In Hössinger et al. (2019) the FOC are given as:

$$\frac{\theta_w}{T_w} + \lambda w - \mu = 0 \tag{12}$$

$$\frac{\theta_i}{T_i} - \mu = 0, \forall i \in A_f \tag{13}$$

$$\frac{\phi_j}{E_j} - \lambda = 0, \forall j \in G_f \tag{14}$$

By solving them, one gets expressions for μ and λ:

$$\mu = \frac{\partial U}{\partial T_i} = \frac{\Theta}{\tau - T_w - T_c} \qquad \lambda = \frac{\partial U}{\partial E_j} = \frac{\Phi}{wT_w - E_c} \tag{15}$$

Actually, Φ and θ_w are the only parameters needed for the calculation of VoL and VTAW. Therefore, one could only estimate the labor supply defined in Eq. (7), but the estimates would be less efficient as the additional information would be ignored (Eq. 8 and 9). Many of the models based on the presented modeling framework and estimated prior to Hössinger et al. (2019) and Jokubauskaitė et al. (2019) did not use the expenditure equation (Eq. 9). As shown in Jokubauskaitė et al. (2019), the inclusion of expenditure estimation might play a crucial role. Without it, the VoL was overestimated by around 20% for Austria. The lack of other comparable estimations might be caused by the almost non-existent availability of time-use surveys with expenditure information. The MAED survey used in this study and carried out in Austria in 2015 is an exception.

3 Application

3.1 MAED data

The estimation of Eq. (7) - (9) requires detailed information not only on time-use (activities), but also on expenditures. To facilitate this, the Mobility-Activity-Expenditure-Diary (MAED) was developed (Aschauer et al., 2018). The survey was carried out in the form of self-administered mail-back questionnaires with telephone support and incentives in Austria in spring and autumn of 2015. Participants reported their data for one week. This period was chosen as a trade-off between the hardship of response and proper portrayal of the individual's long-term equilibrium. Diary-type information on all travel and non-travel activities as well as on any expenditures was gathered simultaneously (from the same individual at the same time) including 10 activity types, 14 expenditure groups, and 5 transport modes (walk, bike, car driver, car passenger, public transport). This makes MAED data one of a kind.

748 representatively selected Austrian workers[6] took part in the survey. Hössinger et al. (2019) present a detailed review of plausibility checks, imputations, and error corrections done to the data. Aschauer et al. (2019) and Aschauer et al. (2018) give a comprehensive overview of MAED characteristics.

Overall, the MAED data are fairly consistent with the latest Austrian Time Use Survey (ATUS; 2008/09) as well as the Austrian Consumer Expenditure Survey (ACES; 2009/10) data (see Tables 2 and 3). Noticeable variations appear in expenditures on housing (ACES - 16.53 % and MAED - 22.15 %, Table 2) and savings (ACES - 11.01 % and MAED - 4.38 %, Table 2). As Hössinger et al. (2019) explains, "the original ACES data include rental equivalents (instead of reported expenses) of owner-occupied housing. The MAED data include, in contrast, only reported mortgage repayments and operating costs, which are not comparable to the (on average lower) rental equivalents. Since we found no way to make both procedures comparable, we removed the rental equivalents in the ACES, which explains the lower share." In addition, there was an actual drop in the saving rate. According OeNB (2018) the saving rate of private households has dropped from 11.4% to 6.8 % between 2009 and 2015. Another divergence is the travel time. In the MAED sample, individuals devote more time to travel than in ATUS (1.23 hr/week, Table 3). This might be more of a problem in ATUS than in the MAED, as time-use surveys are prone to under-report travel (Gerike et al., 2015; Aschauer et al., 2018). What is more, there is a shift in personal care and leisure, caused by the overlapping nature of such activities. In the MAED, individuals identified activities as leisure if they were performed of one's own free will. As Hössinger et al. (2019) point out, some activities (e.g., going to church, visiting the sick) might be classified as leisure in ATUS, while in MAED they are assigned to "Personal", as people did not perceive them as leisure.

In terms of socio-economic segments, the biggest differences between segments in MAED expenditure categories seem to occur with respect to expenditures for housing and food. Otherwise, expenditure shares do not vary greatly within and between the segments. In contrast, time-use allocation (Table 3) varies substantially between socio-economic segments The biggest difference in time allocation by segment is in terms of the time spent on paid work and domestic work. On average, per week, women spend around nine hours less in paid work and eight hours more in the unpaid/domestic work than men (Table 3, Table 5).

[6]Only workers were chosen because information on wage rates as well as work times is needed for the estimation.

Table 2 Distribution of expenditure in different segments, %

		Housing	Food	Mobile	Savings	Leisure	Accomodation	Clothing	Furniture
Global	ACES 2009/10	16.53	15.1	11.56	11.01	9.91	5.52	5.43	5.29
	MAED	22.15	16.59	12.10	4.38	7.42	5.49	5.38	2.30
Urbanity	= urban	23.06	16.28	10.39	4.67	7.69	7.10	5.54	2.32
	= nonurban	21.84	16.69	12.68	4.28	7.34	4.94	5.33	2.29
Gender	= male	21.85	17.07	12.16	4.20	7.40	5.51	5.40	2.34
	= female	22.57	15.91	12.02	4.64	7.46	5.46	5.36	2.24
Age	< 46 years	23.64	17.38	11.69	2.71	7.24	5.71	5.86	2.31
	>= 46 years	21.01	15.98	12.42	5.66	7.57	5.32	5.02	2.29
Education	< HS degree	21.12	18.26	12.72	3.92	6.93	4.70	5.61	2.48
	>= HS degree	22.66	15.76	11.79	4.61	7.67	5.88	5.27	2.21
Children	= no	21.22	15.37	11.81	7.13	7.61	5.42	5.28	2.21
	= yes	23.79	18.74	12.62	-0.47	7.10	5.60	5.56	2.46
No. of workers	= 1	25.24	16.25	12.04	1.57	7.22	5.67	5.23	2.15
	>= 2	21.17	16.69	12.12	5.27	7.49	5.43	5.43	2.35
Pers. income	< 432 eur/wk	24.12	18.80	13.41	-2.81	7.10	5.59	5.88	2.53
	>= 432 eur/wk	21.19	15.51	11.47	7.90	7.58	5.44	5.14	2.19

		Services	Insurance	Electricity	Health	Other	Education	Finance
Global	ACES 2009/10	5.08	4.84	3.82	2.81	1.91	1.14	0.05
	MAED	2.99	7.81	3.48	2.31	4.47	1.96	1.18
Urbanity	= urban	2.83	6.54	3.91	2.08	4.01	2.47	1.11
	= nonurban	3.05	8.23	3.33	2.39	4.62	1.78	1.20
Gender	= male	2.83	8.11	3.50	2.28	4.31	1.87	1.17
	= female	3.21	7.38	3.44	2.35	4.69	2.08	1.18
Age	< 46 years	2.89	7.68	3.33	2.14	4.28	1.97	1.19
	>= 46 years	3.07	7.91	3.59	2.44	4.61	1.94	1.17
Education	< HS degree	2.94	8.44	3.51	1.95	4.51	1.78	1.13
	>= HS degree	3.02	7.49	3.46	2.49	4.44	2.05	1.20
Children	= no	3.00	7.73	3.61	2.18	4.37	1.88	1.19
	= yes	2.97	7.94	3.25	2.53	4.64	2.10	1.17
No. of workers	= 1	3.28	7.82	3.43	2.47	4.44	1.91	1.26
	>= 2	2.90	7.80	3.49	2.26	4.47	1.97	1.15
Pers. income	< 432 eur/wk	3.15	8.08	3.97	2.28	4.79	1.84	1.30
	>= 432 eur/wk	2.91	7.67	3.24	2.33	4.31	2.01	1.12

Table 3 Distribution of activities in different segments, hr/week

		Eating	Travel	Personal	Sleep	Work
Global	ATUS 2008/09	8.63	8.05	6.77	55.88	36.87
	MAED	9.29	9.28	9.54	55.08	37.84
Urbanity	= urban	9.45	9.76	10.30	55.31	39.16
	= nonurban	9.25	9.12	9.30	55.01	37.42
Gender	= male	9.53	9.92	9.03	54.19	42.34
	= female	9.06	8.63	10.06	55.97	33.35
Age	< 46 years	9.26	9.01	9.44	55.47	35.90
	>= 46 years	9.33	9.53	9.64	54.71	39.67
Education	< HS degree	8.76	8.74	9.06	55.21	37.65
	>= HS degree	9.64	9.62	9.85	55.00	37.96
Children	= no	9.09	9.24	9.36	55.13	39.26
	= yes	9.65	9.35	9.86	55.00	35.37
No. of workers	= 1	8.98	9.06	9.37	55.17	40.92
	>= 2	9.38	9.33	9.59	55.06	37.00
Pers. income	< 432 eur/wk	9.26	9.03	10.11	56.06	32.78
	>= 432 eur/wk	9.33	9.53	8.96	54.08	43.05
		Leisure	Domestic	Shopping	Other&Misc.	Education
Global	ATUS 2008/09	32.32	14.93	1.98	1.52	1.05
	MAED	28.93	14.27	2.05	0.41	1.29
Urbanity	= urban	26.98	12.56	2.24	0.33	1.90
	= nonurban	29.56	14.82	1.99	0.44	1.10
Gender	= male	29.78	10.11	1.65	0.37	1.08
	= female	28.09	18.42	2.46	0.45	1.51
Age	< 46 years	28.95	15.41	2.14	0.44	1.97
	>= 46 years	28.92	13.20	1.97	0.39	0.65
Education	< HS degree	30.40	14.63	2.18	0.34	1.03
	>= HS degree	27.99	14.05	1.97	0.46	1.46
Children	= no	29.63	12.29	2.02	0.42	1.57
	= yes	27.74	17.70	2.11	0.40	0.82
No. of workers	= 1	28.75	12.15	2.15	0.44	1.00
	>= 2	28.98	14.85	2.02	0.40	1.37
Pers. income	< 432 eur/wk	29.73	16.56	2.31	0.39	1.78
	>= 432 eur/wk	28.12	11.92	1.78	0.43	0.79

To estimate the model (in particular Eq. 7 - 9), activities and expenditures had to be separated into freely chosen and committed ones. Aggregation results are presented in Table 4. Unlike Hössinger et al. (2019) or Jokubauskaitė et al. (2019), we use three expenditure equations $\{E_{f1}, E_{f2}, E_{f_3}\}$ and thus we get estimates for $\{\phi_1, \phi_2\}$ and ϕ_3 is calculated as $1 - \phi_1 - \phi_2$. This is done in order to take into account available information on the savings (S), which was previously only implicitly included into the estimation as part of E_{f2}. As Schmid et al. (2020) correctly points out, the estimated exponent ϕ_2 from the previous studies was related to the estimation of $E_{f2} + S$ and not purely to E_{f2}. For other groups, the aggregation remained the same as in Hössinger et al. (2019) and Jokubauskaitė et al. (2019). More information on the grouping of variables can be found in Hössinger et al. (2019), Jokubauskaitė et al. (2019),Aschauer et al. (2019), Jara-Díaz et al. (2016), and Mokhtarian and Chen (2004).

Table 4 Shares and correlations of expenditure and time-use data

Activities			Expenditure			
	Var.	%		Var.	%	
Observed categories:						
Paid work	T_w	22.52	Leisure	E_{f1}	7.42	
Leisure	T_{f1}	17.22	Accomm	E_{f1}	5.49	
Eating	T_{f2}	5.53	Electronic	E_{f1}	3.48	
Shopping	T_{f2}	1.22	Clothes	E_{f2}	5.38	
Unspec	T_{f2}	0.06	Savings	E_{f3}	4.38	
Sleep	T_c	32.79	Housing	E_c	22.15	
Unpaid work	T_c	8.50	Food	E_c	16.59	
Personal	T_c	5.68	Mobility	E_c	12.10	
Travel	T_c	5.52	Insurance	E_c	7.81	
Education	T_c	0.77	Other	E_c	4.47	
Other	T_c	0.18	Service	E_c	2.99	
			Health	E_c	2.31	
			Furniture	E_c	2.30	
			Education	E_c	1.96	
			Financing	E_c	1.18	
Model variables:						
	T_w	22.52		E_{f1}	16.39	
	T_{f1}	17.22		E_{f2}	5.38	
	T_{f2}	6.81		E_{f3}	4.38	
	T_c	53.44		E_c	73.85	

Correlations:

	T_w	T_{f1}	T_{f2}	T_c	E_{f1}	E_{f2}	E_{f3}
T_{f1}	-0.22***						
T_{f2}	-0.18***	-0.19***					
T_c	-0.60***	-0.58***	-0.04				
E_{f1}	0.39***	-0.12***	0.11**	-0.26***			
E_{f2}	0.26***	-0.03	0.12**	-0.23***	0.31***		
E_{f3}	0.19***	0.03	-0.10**	-0.15***	0.41***	0.25***	
E_c	0.57***	-0.17***	-0.05	-0.33***	0.56***	0.36***	0.45***

Signif. codes: *** p < .001, ** p < .01, * p < .05

In the MAED data, the average share of time spent on paid work is 22.52% and to unpaid work 8.50%, respectively. On average, committed activities (T_c) make up 53.44% of the total time budget and committed expenditures (E_c) 73.85% of the monetary budget. No assumptions about relationships between goods and activities are made, as the Jara-Díaz and Guevara (2003) model does not require a one-to-one mapping between them. Most empirical correlations are statistically significant (Table 5) and will be taken into account while estimating the model with maximum likelihood (ML). In addition, the aggregated modeling variables/groups are broadly defined, thus there is no danger of running into corner solutions (zero values).

The gender-specific distribution of activities can be seen in Table 5. Only domestic work and paid work exhibit strong gender-specific differences.

Table 5 Distribution of activities by gender, hr/week

Activity	Variable	Men	Women	Δ
Paid work	T_w	42.34	33.35	**8.99**
Unpaid work	T_c	10.11	18.42	**-8.31**
Shopping	T_{f2}	1.65	2.46	-0.81
Eating	T_{f2}	9.53	9.06	0.47
Leisure	T_{f1}	29.78	28.09	1.69
Travel	T_c	9.92	8.63	1.29
Sleep	T_c	54.19	55.97	-1.78
Personal	T_c	9.03	10.06	-1.03
Other&Misc.	T_c	0.37	0.45	-0.08
Education	T_c	1.08	1.51	-0.43

3.2 Valuation of domestic work

Several possibilities exist for the valuation of domestic work, including the opportunity cost method (OCM; Luxton, 1997) and the replacement cost method (RCM; Folbre, 2006). The first approach is based on the forgone value of paid employment, while the second one is based on the cost of hiring somebody to carry out the domestic work. The latter approach can be further divided into specialist (RCM-S), generalist (RCM-G), and hybrid (RCM-H) approaches. The specialist approach takes the cost of hiring a specialist to do some specific work, while the generalist approach takes an 'all-around' worker (such as housekeeper). RCM-H is a mix between the previous two approaches, where household activities are priced according to the generalist approach and more complex tasks (such as child or elderly care) are priced in accordance with the rate demanded by a specialist. The latter approach has been applied to, among others, data in Mexico, Canada, and Australia (UNECE, 2017). Depending on which approach is chosen, the value of domestic work may vary considerably. In Jackson and William (2015), the share of unpaid work in the GDP was reported to vary between 35% and 70% when employing the OCM and between 35% and 60% with an RCM-S approach. A similarly strong variation was reported in Schmid et al. (2002), where the OCM resulted in a valuation of unpaid domestic work equal to 37.5% of GDP, while the RCM-S approach showed domestic work being valued at 57.9% of GDP in Switzerland. The authors also tried to compare Switzerland, Australia, Finland, the Netherlands, Canada and Norway, but concluded that due to the differences in data gathering (time-use survey), the comparison might not be meaningful. Reports from UNSD (2005), UN (2014), and Eurostat (2008) have proposed guidelines for the harmonization of time-use surveys and UNECE (2017) made a recommendation to use the replacement cost generalist approach, meaning to use the wage rate of an "all-round" worker/housekeeper.

In this paper, we take an approach similar to the RCM-G, but we do not use the national account (NA) data. Instead, we use information available on the Internet, which should depict the current market situation for domestic work better. We have extracted data from two Internet platforms, where people (not companies) offer their services to perform domestic work. We will call them "Platform A" and "Platform B". For the robustness check, data from both platforms are analyzed. On the first website, the following work categories are available: cleaning services (cleaning, ironing ...), gardening, craft-works, property management during absence, and combinations of these tasks. One person can offer more than one service and the listed price is the average of all offered categories. Thus, there is no clear separation between the prices of different services. One can only get information on so-called "all-rounders" willing to do some or all predefined tasks. In the second platform, individuals offer their work for distinct services. This allows for a clear identification of service-specific wage rates. In comparison to the first platform, one can find a wage rate for house & garden (a combination of cleaning, gardening, craft-works), childcare, elderly-care, pet-care, but no combinations for the same user. In each platform, employers can get the following information on the worker (if provided by the worker): postal code, age, years of experience, requested wage rate. In addition to this, "Platform A" makes it possible to filter workers by gender and spoken languages. New users are added each day to these platforms and prices are negotiable. The data was collected in August 2018[7]. Average prices of services by region across Austria are presented in Fig. 2.

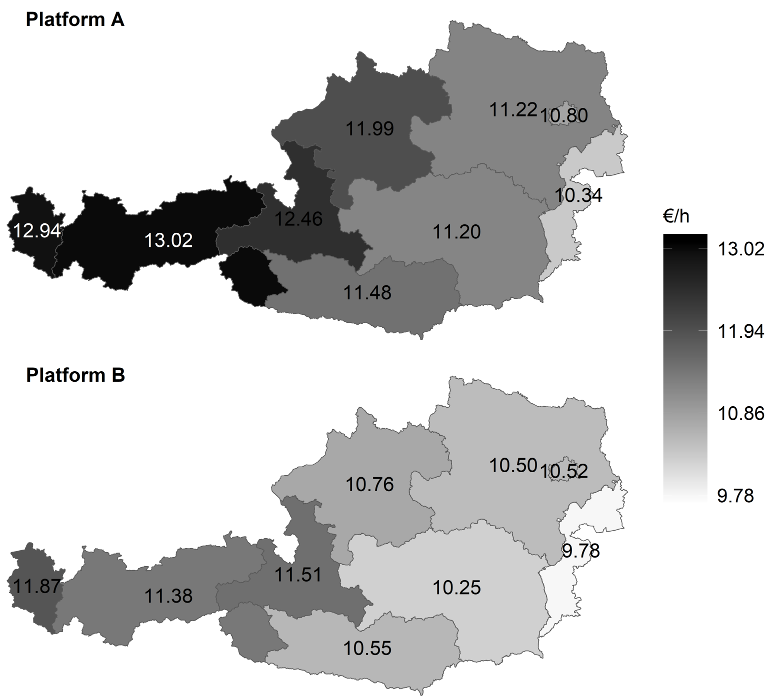

Fig. 2 Average wage rate of offered services by regions, €/hr

Table 6 Average wage rate by service and federal state, €/hr

	Platform B				Platform A			
	Total	House&garden	Childcare	Elderly-care	Total	Gardening	Craftworks	House
Austria	10.62	10.92	10.23	11.19	11.28	11.32	11.60	11.25
Burgenland	9.78	9.78	9.53	10.42	10.34	10.43	10.76	10.31
Styria	10.25	10.53	9.82	10.86	11.20	11.12	11.16	11.18
L. Austria	10.50	10.89	9.94	11.20	11.22	11.25	11.68	11.15
Vienna	10.52	10.53	10.41	10.98	10.80	10.90	11.00	10.79
Carinthia	10.55	10.95	9.92	11.61	11.48	11.49	11.89	11.47
U. Austria	10.76	11.18	10.10	11.63	11.99	12.04	12.63	11.92
Tyrol	11.38	12.46	10.68	11.43	13.02	13.46	13.94	12.98
Salzburg	11.51	12.11	10.90	12.00	12.46	12.11	12.24	12.46
Vorarlberg	11.87	12.73	11.05	12.51	12.94	12.21	12.32	12.97

The data collected from "Platform A" imply higher wage rates than that from "Platform B", due to the way in which the wage on each platform is specified. On "Platform B", people specify the minimal wage rate, while on "Platform A" workers list their desired hourly rate ("gewünschter Stundensatz"). Thus, the data from "Platform B" underestimate the actually paid wage rate and those from "Platform A" overestimate it. Both data sets exhibit a wage increase from the east to the west of Austria (Fig. 2). This might be partly explained by the spillover effects of lower wage rates of neighboring countries as well as by higher

[7]Wages for pet-care were excluded from "Platform A" as almost all listings implied wage rates of around €7/hr, strongly weighing down the average wage rate across different types of tasks.

unemployment in the eastern regions. In 2017, of the nine Austrian regions, Vienna (in the East) had the highest unemployment rate (13.0%) and Vorarlberg (in the West) had one of the lowest (5.8%) (AMS, 2017). To the east, Austria is surrounded by low wage countries - the Czech Republic, Slovakia, Hungary and Slovenia - and in the west by high-income countries - Italy, Switzerland, and Germany. In addition, Vienna as by far the largest city of Austria has the highest immigrant share (ÖIF, 2018). This fact, in combination with the higher unemployment rate in Vienna, might explain the lower observable wage rate in this state. On "Platform B" the most expensive service is "Elderly-care", and on "Platform A" it is "Craftworks" (Table 6). In the "Platform A" sample, around 90% of observations are identified as female. As "Platform B" does not offer gender-specific data, the gender had to be predicted. After name matching and comparison, 91% of observations were identified as female. Thus, both data sets are strongly female dominated.

Although the "Platform B" dataset might undervalue housework, it provides information on childcare, which consumes a lot of time for households with children. Thus, the "Platform B" data was ultimately chosen for the estimation. According to Ghassemi et al. (2009), in Austria working individuals spend on average around 19.5 hr/week doing housekeeping and childcare as a primary activity. Childcare is also often done as a secondary activity: around 8% of women reported almost one hr/day of talking with a child as a secondary activity, 4% of women and 2%of men played with children while being involved in another activity (Ghassemi et al., 2009). According to the same source, working men spend on average 11.4 hr/week engaging in housekeeping tasks, whereas women do almost twice that (21.6 hr/week). The difference in terms of the average time assigned to childcare is only one hr/week (women - 4.2 hr/week and men - 3.2 hr/week [8]).

The next step is to merge the MAED data with the "Platform B" data. As both data sets were gathered in different years (MAED in 2015, "Platform B" in 2018), the wage rates collected from "Platform B" had to be adjusted for inflation. According to Statistik Austria, the yearly inflation with basis 2015 in 2018 was 5.1%. Next, data matching between MAED and "Platform B" had to be done. From each data set it is possible to identify where individuals live according to the four-digit postal code. These areas can be further spatially aggregated to obtain larger areas approximating the nine provinces of Austria. The two data sets (MAED and "Platform B") were merged by two-digit postal codes, which carry information on the province in which an individual lives. Merging by larger spatial area was necessary in some cases, because otherwise we would lack reliable information on some of the service types. Then for each individual included in the MAED data, filtering by four-digit postal code was done. If no "Platform B" data from the same postal code area was available, information from the two-digit postal code area (i.e. the provincial level) was used. Next, it was checked whether a person from the MAED data has children: if yes, observations from "Platform B" having the same postal region (or two-digit postal area) that offer services of childcare were identified. To get the closest match ("best approximation") from "Platform B" for the MAED individual, the search was narrowed down by choosing individuals from "Platform B" of the same gender as the MAED individual.

For the final calculation of the domestic wage rate, two chores were used: housekeeping (house & garden) and childcare. The MAED data does not distinguish between different types of domestic work. Thus, proportions for housekeeping and childcare were chosen based on the Austrian Time Use Survey (ATUS) and considering only these two household chores (Table 7). According to these proportions, working women devote 84% of domestic work[9] to housekeeping and 16% to childcare, whereas for men these amount to 78% and 22%, respectively. The average total chore split is 82% (housekeeping) and 18% (childcare). In order to test the robustness of our results with respect to different weights/ratios that unpaid work is assumed to be composed of, a sensitivity analysis was performed. Three different splits were used: 50%-50%, 82%-18% (total split), gender-specific (women split 84%-16%, men split 78%-22%). These weights can help to approximate the domestic wage rate - w_D which is specific to a given individual in the MAED dataset.

Table 7 Distribution of domestic work according to the Austrian Time Use Survey, %

Service	Women	Men	Total
Housekeeping	83.72	78.08	81.54
Childcare	16.28	21.92	18.46

[8] Average was calculated for the whole population and not for the people with children.
[9] Here domestic work is defined only as a sum of housekeeping and childcare.

Besides treating domestic work as official (paid) work, we do not make any further changes in the theoretical model. We make the following five data adjustments for each split of household chores separately:

0. Calculate the average domestic wage rate: $w_D = w_{\text{housekeeping}} w_1 + w_{\text{cildcare}} w_2$, $w_1 \in \{50, 82, gender\}$, $w_2 \in \{50, 18, gender\}$.
1. Synergy of official and domestic work wage rates: $w^{new} = (wT_w + w_D T_D)/(T_w + T_D)$.
2. Treat domestic work as official work: $T_w^{new} = T_w + T_D$.
3. Balance the time constraint: $T_c^{new} = T_c - T_D$.
4. Balance the budget constraint: $E_c^{new} = E_c + w_D T_D$.

Table 8 Summary statistics according to different data adjustment procedures

	Original					50%-50%			
	Global	Men	Women	Δ		Global	Men	Women	Δ
w	12.14	12.76	11.51	1.25	w^{new}	11.74	12.46	11.02	1.44
w_D						10.52	10.78	10.27	0.51
T_w	37.84	42.34	33.35	8.99	T_w^{new}	52.10	52.44	51.77	0.67
T_c	89.77	84.58	94.95	-10.36	T_c^{new}	75.51	74.49	76.53	-2.04
E_c	332.44	394.74	270.32	124.42	E_c^{new}	481.74	503.62	459.91	43.72
$wT_w - E_c$	127.49	148.06	106.97	41.09		-	-	-	-
$\tau - T_w - T_c$	40.39	41.08	39.71	1.37		-	-	-	-
	82%-18%					Gender-specific			
	Global	Men	Women	Δ		Global	Men	Women	Δ
w^{new}	11.77	12.49	11.06	1.43		11.77	12.49	11.06	1.42
w_D	10.65	10.94	10.36	0.58		10.64	10.92	10.36	0.56
E_c^{new}	483.70	505.44	462.03	43.41		483.68	505.24	462.17	43.06

Results of the data adjustments are presented in Table 8. The domestic wage rate is lower than the wage rates paid on the official labor market; as a result, the “new” official wage rate is now lower for both genders. The female wage rate drops more, as women work relatively more in the domestic sector. Correspondingly, the wage gap increases from €1.25/hour to €1.4/hour (Table 8) when we include the valuation of unpaid work. After these adjustments, men work in total around one hr/week more than women do (row T_w^{new}). There is no significant difference between the “gender-specific” and the “82%-18%” split. Thus, in addition to the “50%-50%” split, only the “82%-18%” data adjustment, being the easier one, will be used in the further analysis. The question now is how these will affect the VoL, which was previously estimated to be €11/hour for men and €6/hour for women in Jokubauskaitė et al. (2019).

3.3 Estimation

The theoretical model (Eq. 7 - 9) is estimated as a system of g nonlinear equation with contemporaneous covariance matrix Σ:

$$Y_i = g_i(\beta) + \eta_i, \quad \mathbf{E}(\eta_i \eta_j^\top) = \sigma_{ij} \mathbf{I}_n, \quad i, j = \{1, \ldots, g\}. \tag{16}$$

Here i, j are equation specific indices. The dependent variable Y_i is modeled through the regression function g_i and error term η_i, which is assumed to be normally distributed ($\eta_i \sim N(\mu_i, \sigma_i)$). β is a vector of parameters to be estimated, σ_{ij} is an element from the covariace matrix Σ and I_n is an $n \times n$ identity matrix. Estimation results[10] can be found in Table 9.

[10] The estimation of the nonlinear system of equations is based on the R package *maxLik* (Henningsen and Toomet, 2011) and self-written R package *nmm*. The latter package allows flexible estimation of Nonlinear Multivariate Models (NMM) and will be published on CRAN upon completion.

The first model is the one estimated with the original data and modeling framework, where domestic work is treated as committed activity with no wage rate attached to it. The other model estimations presented in Table 9 correspond to different wage rates attached to domestic work. As both data adjustments ("50%-50%" and "82%-18%") result in almost identical estimates and "82%-18%" represents the real world situation more accurately, only the "82%-18%" split will be used in the following analysis.

There is almost no change across the models in the estimated parameter θ_1, which shows the elasticity of utility with respect to the first freely chosen activities group, and minimal changes in log-likelihood, adjusted R^2, and correlations. The FOC given by Eq. (13) defines the binding relationship between $\{\theta_i, T_i, \mu\}$. Thus, if T_i does not change and θ_i does not change, the marginal utility of leisure time (μ) remains constant. And if μ does not change and disposable income as well as time remain constant, the change in the VoL is caused by either λ or θ_w (Eq. 10 and 11, Table 10). In the further analysis, we will concentrate on the FOC given by Eq. (12)-(14). Eq. (12) consists of three marginal utilities: work-time, money and leisure time.

According to the estimation results (Table 9), $\{\Phi, \phi_1, \phi_2\}$ decrease about 20%-30 %. To keep the Eq. (14) valid, λ has to decrease proportionally to this change. This decrease in turn leads to an increase in the VoL (Table 10, column "Global"). With a decreasing wage rate and a decreasing λ, the marginal utility of money (λw) declines. The model tries to compensate the loss and equilibrate Eq. (12) by raising the marginal utility of work (θ_w/T_w). Because T_w increases, θ_w has to go up. Although, in comparison to the original model the estimate of θ_w almost halves, it remains statistically significant. As a result, the VTAW also almost halves, but remains negative; the marginal time spent at work generates dis-utility, but not as large as with the original data. In the new setting, work is perceived more positively and leisure is more valuable.

Table 9 Estimation results

	Original		50%-50%		82%-18%	
	Par	s.e	Par	s.e	Par	s.e
θ_w	-0.602	(0.086)	-0.337	(0.074)	-0.329	(0.074)
Φ	0.413	(0.027)	0.323	(0.019)	0.320	(0.019)
θ_1	0.733	(0.004)	0.732	(0.004)	0.732	(0.004)
ϕ_1	0.260	(0.017)	0.203	(0.012)	0.201	(0.012)
ϕ_2	0.085	(0.006)	0.066	(0.004)	0.066	(0.004)
σ_{Tw}	62.447	(0.153)	62.278	(1.595)	62.228	(1.653)
σ_{Tf1}	68.379	(0.758)	68.718	(1.772)	68.716	(1.856)
σ_{Ef1}	39.204	(0.540)	40.671	(1.040)	40.773	(1.062)
σ_{Ef2}	20.394	(0.538)	20.564	(0.535)	20.562	(0.537)
$\rho_{Tw\&Tf1}$	-0.696	(0.016)	-0.701	(0.019)	-0.701	(0.019)
$\rho_{Tw\&Ef1}$	0.343	(0.030)	0.397	(0.031)	0.400	(0.031)
$\rho_{Tw\&Ef2}$	0.127	(0.036)	0.163	(0.036)	0.165	(0.036)
$\rho_{Tf1\&Ef1}$	-0.450	(0.027)	-0.483	(0.028)	-0.484	(0.028)
$\rho_{Tf1\&Ef2}$	-0.218	(0.035)	-0.244	(0.035)	-0.245	(0.035)
$\rho_{Ef1\&Ef2}$	0.090	(0.036)	0.122	(0.036)	0.123	(0.036)
LL_{null}	-17170.083		-17136.798		-17135.453	
LL_{final}	-12214.024		-12223.104		-12223.040	
$R^2_{adj}: T_w$	0.691		0.687		0.687	
T_{f1}	0.619		0.615		0.615	
E_{f1}	0.388		0.336		0.333	
E_{f2}	0.146		0.135		0.136	
Wage	12.136		11.737		11.774	
VoL	7.191	(0.399)	9.171	(0.432)	9.246	(0.435)
VTAW	-4.945	(0.399)	-2.566	(0.432)	-2.528	(0.435)

Signif. codes: *** p < .001, ** p < .01, * p < .05
Standard errors of indicators were calculated using the δ-method proposed in Doob (1935). Adjusted R^2 are calculated using 5-hold cross-validation.

Table (10) shows a summary of gender-specific estimation results and summary statistics. The gender-specific models were estimated using a-priori segmentation (estimation of separate models for different segments). T_w increases for both genders (by 55.90% for women and 24.11% for men) and T_c decreases accordingly. Through this trade-off, T_w and T_c are harmonized within the genders. Due to the balancing of the budget constraint, this change comes along with an analogous change in E_c. As the ratio between disposable income ($wT_w - E_c$) and freetime ($\tau - T_w - T_c$) remains almost the same, only the change in the modeling parameters affects the VoL and VTAW values.

A similar pattern as in the global model is observed for females (Table 10, column "Women"). Their marginal utility (MU) of leisure does not change, their MU of money decreases, and their MU of work-time increases. Due to this, their VoL and VTAW increase. In contrast, for men, the MU of money increases. To compensate this, model tries to decrease θ_w and as a consequence the VTAW decreases. With the adjusted data, men work more than women do and receive higher disutility from work, but because of the higher wage rate, their time is still a little bit "more valuable". Without a huge decrease in the VoL of men, the gender-specific gap in the VoL is closed.

Table 10 Results by gender, estimation based on different data adjustments

	Global			Women			Men		
	Original	82%-18%	50%-50%	Original	82%-18%	50%-50%	Original	82%-18%	50%-50%
VoL	7.19	9.25	9.17	5.52	9.00	8.91	10.12	9.20	9.15
w	12.14	11.77	11.74	11.51	11.06	11.02	12.76	12.49	12.46
VTAW	-4.95	-2.53	-2.57	-5.99	-2.06	-2.11	-2.64	-3.29	-3.31
Φ	0.41	0.32	0.32	0.48	0.29	0.30	0.32	0.36	0.36
θ_w	-0.60	-0.33	-0.34	-0.85	-0.28	-0.29	-0.25	-0.43	-0.43
$1/\Phi$	2.42	3.12	3.10	2.08	3.40	3.36	3.09	2.80	2.78
$\frac{(w\widehat{T}_w - E_c)}{(\tau - \widehat{T}_w - T_c)}$	2.97	2.96	2.96	2.65	2.65	2.65	3.28	3.29	3.29
$w\widehat{T}_w - E_c$	118.09	119.51	119.43	101.26	104.41	104.30	135.26	135.10	135.05
$\tau - \widehat{T}_w - T_c$	40.49	40.33	40.33	39.72	39.51	39.51	41.24	41.16	41.17
$\widehat{T}_w$	37.74	52.16	52.16	33.33	51.96	51.97	42.18	52.35	52.35
E_c	332.44	483.70	481.74	270.32	462.03	459.91	394.74	505.44	503.62
μ	0.00278	0.00273	0.00274	0.00288	0.00279	0.00279	0.00267	0.00268	0.00268
λ	0.00051	0.00033	0.00033	0.00071	0.00033	0.00033	0.00033	0.00034	0.00034
λw	0.00509	0.00340	0.00341	0.00742	0.00335	0.00337	0.00331	0.00353	0.00354
$\frac{\theta_w}{\widehat{T}_w}$	-0.02315	-0.00661	-0.00676	-0.04542	-0.00565	-0.00586	-0.00641	-0.00851	-0.00860

As a plausibility check, additional sensitivity analyses are carried out. In the first one, we assume that each individual in the MAED data obtains the same w_D per hour of domestic work (Fig. 3). The starting point is where individuals receive only €1/hr for domestic work, up to €12/hr. The average wage rate of domestic work from "Platform B" is around €11/hr. In the second analysis, people receive a fraction ranging from 10% to 90% of the offered individual-specific wage rate for domestic work (with housekeeping and child-care distribution "82%-18%", Fig. 4). In both analyses, the gender-specific VoL gap narrows as w_D increases, while the VTAW ($= VoL - w$) remains almost constant.

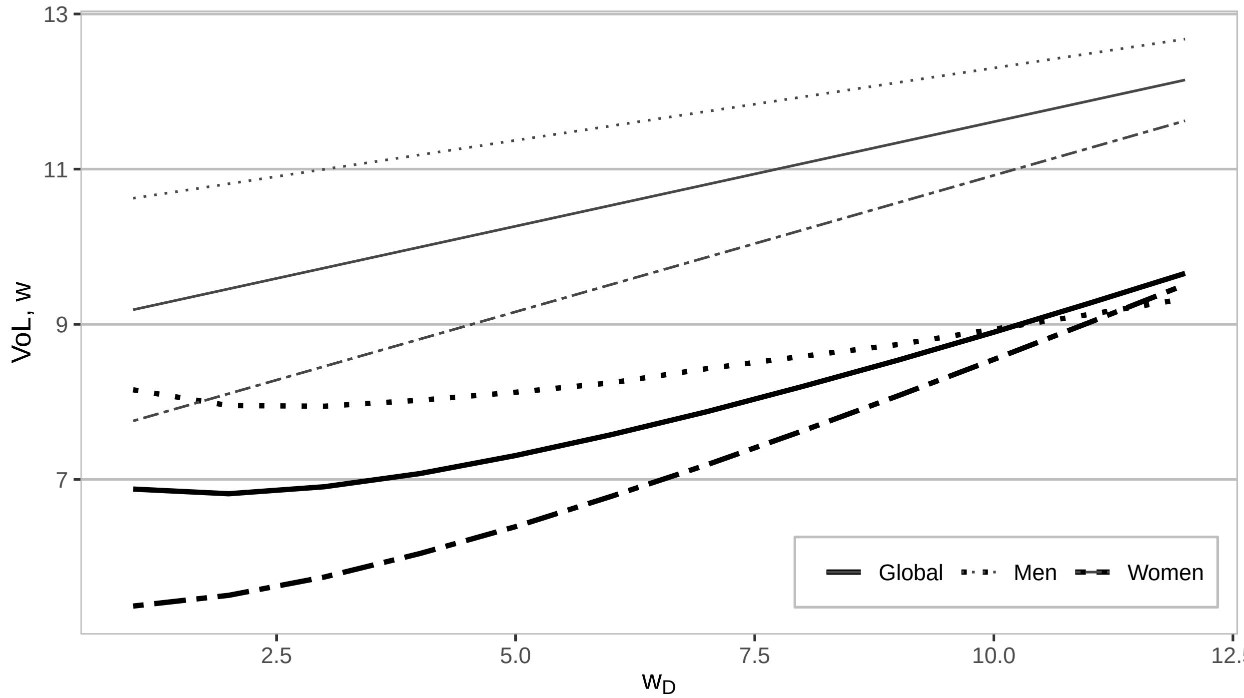

Fig. 3 Simulation results, all get the same wage rate. Thick lines represent VoL, thin - wage rate

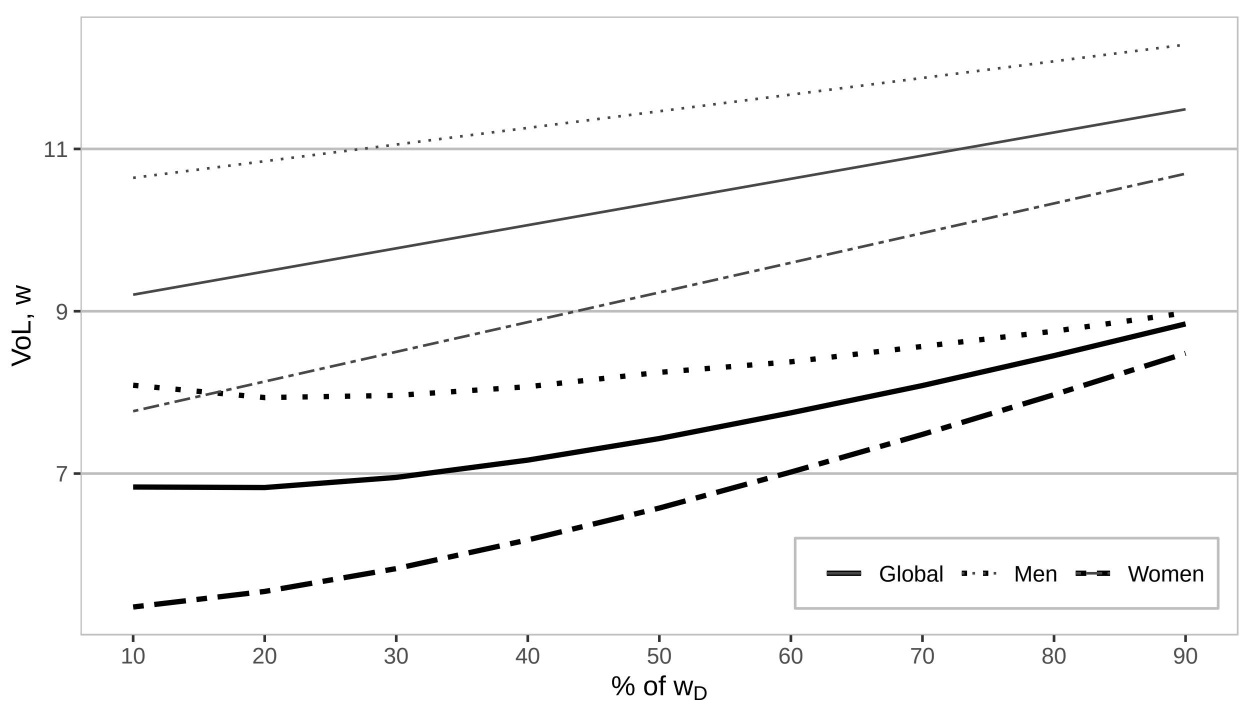

Fig. 4 Simulation results, different % of w_D. Thick lines represents VoL, thin - wage rate

Ahmad and Koh (2011) have estimated that the domestic work in Austria is worth around 24% of GDP. This share is close to the share of GDP contributed by the Austrian industrial sector. The importance of domestic work should therefore not be neglected. The monetarization of domestic work results in an increased estimate of the VoL, which has strong policy relevance. According to Schmid (2019) “Assuming a constant VTTS and decreasing marginal impacts of additional investments on user benefits, a high VoL is reflected in a high VTAT, and investing in speed might be more beneficial (by eventually decreasing the travel time), since the opportunity costs of travel are relatively high, and the conditions of travel are already at a high level (thus leaving less room for improvement).” Other situations are presented in Table 11. Jokubauskaitė et al. (2019) have reported positive values of the VTAT only for public transport. Concluding that they reflect, “the favorable public transport conditions in Austria”. Hence, the updated VoL values might give an even stronger signal for the prioritization of investment in speed rather than comfort of public transport in Austria.

Table 11 Hypothetical examples of policy recommendations when considering the VoL in addition to the VTTS under the assumption of decreasing marginal impacts of additional investments on user benefits.

VTTS[CHF/h]	VoL[CHF/h]	VTAT[CHF/h]	Invest in ...	Implication
20	5	-15	Conditions	VTTS ↓
20	20	0	Cond./Speed	VTTS/travel time ↓
20	35	15	Speed	Travel time ↓

Source: Schmid (2019)

4 Discussion and conclusions

The focus of this study was to research the possible effects of the monetary valuation of unpaid work on the gender-specific value of leisure (VoL) and in turn to offer an updated information set for policymaking based on the VTTS and its components. In many economic models, domestic work is ignored. It usually does not appear in labor statistics as work and it is not factored into the official GDP. One might thus call it the same way as Rollins (1985) did - “invisible work”, which is over-proportionally performed by females, and often associated with lower disposable incomes and as a result lower pensions. In Jokubauskaitė et al. (2019), the VoL of Austrian women is estimated to be worth only half of men’s (€11/hr for men and €6/h for women). We show that this significant difference vanishes when a monetary value is assigned to domestic work. With this adjustment, the VoL was estimated to be around €9/hr for both genders, which is higher than without the adjustment. As the value of travel time savings (the VTTS) increases in the VOL and decreases in the VTAT when the VTAT is estimated to be positive (which has been the case only for public transport in the earlier study based on the same data as this paper, see Jokubauskaitė et al. (2019)), the increase in the VoL reinforces that investments directed towards public transport should prioritize time saving measures over comfort enhancing measures.

The valuation of domestic work might seem to be a complicated task. On the one hand, no “best” method exists; on the other hand, the disregard of its value might lead to gender-biased economic indicators that discriminate against women and their work or anyone who is involved in domestic work. The inclusion of domestic work is important at all stages of economic calculations - from microeconomic models, such as TUM, to the calculation of the GDP. As shown in many research papers on the GDP (Alonso et al., 2019; Bridgman et al., 2018; UNECE, 2017; Payne and Vassilev, 2018; Schwarz and Schwahn, 2016; Ahmad and Koh, 2017; Hamdad, 2003; Schmid et al., 2002), the inclusion of a value for unpaid work might lead up to a 60% increase in GDP, depending on which valuation method is used. For the valuation of domestic work, we have used an approach similar to the replacement cost generalist method commonly used in the re-calculation of GDP. Instead of national account data, we used data available online, which are likely to better reflect the current market situation for domestic work in Austria. Based on location and gender, each MAED individual was matched with the market wage rate for housekeeping and childcare tasks (the latter only for households with children). As both data sets were gathered in different periods, the current market wage rate of domestic work was inflation adjusted. After this modification, the average wage rate of unpaid work (w_D) was computed to be around €10.52/hr (for men €10.78/hr, for women €10.27/hr). Due to the lower w_D for women and higher share of time spent in this type of employment, the new average wage rate (w^{new}) for women decreased more than for men. Despite this, the VoL estimates are similar for both genders: for women it was estimated to be €9.00/hr and for men €9.20/hr. Therefore, we have shown that factoring in the domestic work reduced the gender specific VoL gap, at least for the case of Austria. The applied sensitivity analyses confirmed these results. Also, they do not seem to be overly sensitive to the different types of data adjustments that we have tested.

It is not possible to say which value of the VoL is the "right" one, as the VoL is a latent variable, which cannot be observed directly. Nevertheless, we believe that taking into account that engaging in domestic work means savings in term of expenses is the right way to go. We show that the valuation of domestic work matters and that it is indeed relevant within the framework of microeconomic time use models and time valuation, which in turn is an essential input to appraisals of transport policies and investments.

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Conflicts of interest/Competing interests

Not applicable

Availability of data and material

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Not applicable

Authors' contributions

S. Jokubauskaitė: literature review, data analysis, software development, manuscript writing
R. Hössinger: literature review, data analysis, manuscript writing
S. Jara-Díaz: theoretical model, interpretation, manuscript writing
S. Peer: introduction, conclusions, manuscript writing
A. Schneebaum: literature review, , manuscript editing
B. Schmid: provision of choice data and models, manuscript editing
F. Aschauer: data collection, data description, manuscript editing
R. Gerike: research approach, manuscript editing
K.W. Axhausen: content planning, manuscript editing

F. Leisch: project leader, manuscript editing

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nmm: An R Package for Estimation of Nonlinear Multivariate Models

by Simona Jokubauskaitė, Friedrich Leisch, and Reinhard Hössinger

Abstract Package **nmm** offers a convenient way to estimate a subset of nonlinear multivariate models. It extends the already existing family of packages, which estimate systems of nonlinear regressions and logit models. It uniquely factors in the correlations between the error terms from both models. The estimation is based on **maxLik** with an option to use global optimization via the differential evolution algorithm (**DEoptim**). Instead of using the numerical approximations of gradient and the Hessian, `nmm` produces their analytical versions. The `summary` method of `nmm` object returns "normal", "robust" or "clustered" standard errors. In addition, a wide range of goodness-of-fit statistics is included. The flexible design of logit and system of nonlinear regressions, relaxation of linearity assumption, correlation between continuous and discrete equations allows to use **nmm** in various settings to estimate a wide range of models.

Introduction

The R (R Core Team (2020)) environment provides a rich soil for the estimation of linear systems of equations as well as discrete choice models. One can estimate simultaneous equations using **systemfit** (Henningsen and Hamann, 2007), **thsls** (Taha, 2015) or discrete choice models with **mlogit** (Croissant, 2019), **mnlogit** (Hasan et al., 2016), **MNP** (Imai and van Dyk, 2017), **nnet** (Venables and Ripley, 2002), etc. Things start to get more complicated if the assumption of linearity is dropped and especially if one is working with the multinomial variables. In this paper we introduce a new R package called **nmm** that estimates a subset of non-linear multivariate models (NMM) in the maximum likelihood (ML) framework.

nmm extends the already existing and popular family of packages, that estimate multivariate nonlinear regression (MNR) and logit[1] models. Under the assumption of normality of MNR error terms and by applying the quantile transformation to the probabilities from logit, log-likelihood is formed for the joint system of MNR and logit (Subsection "Joint Continuous-Discrete Model"). By using this strategy, **nmm** package uniquely allows the correlation between the error terms from MNR and the choice probabilities from logit. In addition to this, it enables a very flexible design of multinomial logit. Different indirect utilities can be defined for each alternative separately, one can use a non-linear specifications, and choice set is individual-specific.

The estimation framework is partly based on the **maxLik** package. **maxLik** employs several local ("NR", "BFGS", "BHHH", "CG", "NM") as well as one global ("SANN") optimizer. To enhance this toolbox, **nmm** incorporates the global optimization via the differential evolution algorithm (implemented in **DEoptim** Mullen et al. (2011)). As default, **maxLik** uses the numeric approximations of gradient and Hessian. **nmm** extends this with the usage of analytical counterparts. Only the system of equations, parameter names, and model type is required from the user to produce likelihood, gradient, and Hessian functions. Package supports the estimation of nonlinear simultaneous equations models (NSEM), MNR, logit, and the joint model. The `summary` method of the `"nmm"` object includes standard statistics such as estimates, t-values, probabilities, and standard errors. Latter ones are reported either as "normal", "robust" or "robust-clustered". Additionally, a wide range of goodness-of-fit statistics are included (R^2, BIC, AIC, AICc, etc.) with several pseudo R^2s (McFadden, Cox&Snell, Nagelkerke).

In the following section, we introduce the theory of the NMM estimation. It is divided into estimation of system of nonlinear regressions, nonlinear simultaneous equations, multinomial logit, and the joint estimation of them. Section "Usage of nmm Package" introduces the functionality of **nmm**. The joint estimation is demonstrated with the help of a simple data set and a real world application. The latter one is based on the theoretical model of Munizaga et al. (2008) and a survey data from Austria gathered in 2015. Both the model and the data are presented in Section "Estimation of Theoretical model".

Estimation of Nonlinear Multivariate Models using Maximum Likelihood

Using **nmm** one can estimate several types of NMMs: systems of nonlinear regressions (SNR) also called multivariate nonlinear regressions (MNR), nonlinear simultaneous equation model (NSEM), different types of logit, and the joint continuous-discrete model (JCDM). In this paper, we refer to SNR, MNR, NSEM as the continuous block as the error terms are assumed be normally distributed. Whereas, logit is referred to as discrete block. In this section we introduce theory related to the NMMs. Material from this section is based on the theory on multivariate models presented in MacKinnon and Davidson (1999), Chapter 12.

Systems of Nonlinear Regressions

A system of nonlinear regressions is given by the equation:

$$y_{ti} = x_{ti}(\beta) + u_{ti},\ t = 1, \ldots, n,\ i = 1, \ldots, g \tag{1}$$

Here y_{ti} is the t-th observation of dependent variable i and $x_{ti}(\beta)$ is the t-th observation of the regression function. β is a k-vector of parameters to be estimated and u_{ti} is an error term with an assumption $E(u|X) = 0$. X stands for all explanatory variables that appear in all of the regression functions. Error terms are assumed to be serially uncorrelated, homoscedastic within each equation, and have contemporaneous covariance matrix Σ with elements σ_{ij}:

[1] The dogit model from Gaundry and Dagenais (1979) can be also estimated.

$$E\left(u_{ti}, u_{sj}\right) = \begin{cases} \sigma_{ij}, & t = s \\ 0, & t \neq s \end{cases} \tag{2}$$

If one stacks all observations from variable i vertically, Equation (1) can be rewritten as:

$$y_i = x_i(\beta) + u_i,\ E\left(u_i u_j^\top\right) = \sigma_{ij} I_n,\ i, j = 1, \ldots, n \tag{3}$$

Under the normality assumption, the log-likelihood function is:

$$LL(\beta) = \log(L(\beta)) = -\frac{gn}{2} log2\pi - \frac{n}{2} log|\Sigma| - \frac{1}{2}\left(y_{.} - x_{.}(\beta)\right)^\top \left(\Sigma^{-1} \oplus I_n\right)\left(y_{.} - x_{.}(\beta)\right) \tag{4}$$

Here $y_{.}$ denotes gn-vector of y_is stacked vertically, $x_{.}(\beta)$ denotes gn-vector of $x_i(\beta)$s stacked in the same way. $LL(\beta)$ is maximized with respect to β for a given Σ. The estimated variance-covariance matrix of error terms based on the maximum likelihood estimation (MLE) can be found with the formula:

$$\hat{\Sigma}_{ML} = \frac{1}{n} U^\top\left(\hat{\beta}_{ML}\right) U\left(\hat{\beta}_{ML}\right) \tag{5}$$

Here $U\left(\hat{\beta}_{ML}\right)$ is $n \times g$ matrix with columns equal to $y_i - x_i\left(\hat{\beta}_{ML}\right)$.

Nonlinear Simultaneous Equation Model

In the case of nonlinear simultaneous equation model (NSEM), one has g equations with g endogenous variables. Equations can be expressed as:

$$f_{ti}(Y_t, \beta) = u_{ti},\ t = 1, \ldots, n,\ i = 1, \ldots, g \tag{6}$$

Here functions f_{ti} depend on the predetermined variables and are in general nonlinear functions of both Y_t ($1 \times g$ vector) and β (k-vector).

$f_i(Y, \beta)$ is n-vector with element t equal to $f_{ti}(Y_t, \beta)$ and Y is the $n \times g$ matrix with row t equal to Y_t. We can stack $f_i(Y, \beta)$ horizontally to get $h_t(Y_t, \beta)$, which is $1 \times g$ row vector containing the elements $f_{t1}, \ldots, f_{tg}$. Now Equation (6) can be rewritten as:

$$h_t(Y_t, \beta) = U_t,\ \ U_t \sim \mathcal{NID}(0, \Sigma) \tag{7}$$

Model from Equation (7) has the following concentrated log-likelihood function:

$$-\frac{gn}{2}(log2\pi + 1) + \sum_{t=1}^{n} log\left|detJ_t\right| - \frac{n}{2} log\left|\frac{1}{n}\sum_{t=1}^{n} h_t^\top(Y_t, \beta) h_t(Y_t, \beta)\right| \tag{8}$$

Here $J_t = \partial h_t(\beta) / \partial Y_t$ is different $\forall t$.

Multinomial logit

In the logit model, individual maximizes her/his utility by making a choice (q), which generates the maximum utility:

$$U_q = V_q + \epsilon_q \geq max_{m \neq q}\{U_m\} \tag{9}$$

$$V_q \geq max_{m \neq q}\{U_m\} - \epsilon_q = \omega_q \tag{10}$$

Utility U consists of the observable part V (indirect utility) and an error term ϵ. Under the assumption that ϵ_q terms are Gumbel distributed, ω_q follows the logistic distribution (Domencich et al., 1975).

If V is a function of parameter vector θ, the distribution function of V_q can be expressed as:

$$P_q(\theta) = F(V_q(\theta)) = \frac{exp(V_q(\theta))}{\sum_{j=1}^{Q} \alpha_j exp(V_j(\theta))} \tag{11}$$

Here Q is a set of possible choices, α_j is a dummy variable equal to one if alternative is available and zero otherwise. If repeated measurements are allowed, the corresponding log-likelihood function can be written as:

$$LL\left(\theta\right) = \sum_{i=1}^{J} \sum_{k \in A_i} \sum_{q=1}^{Q} \delta_{ikq} \alpha_{ikq} \log\left(P_{ikq}\right) \tag{12}$$

Here J is number of individuals or index set, A_i is a set of repeated choices for individual i. δ_{ikq} is equal to one if alternative q was chosen by individual i at choice k.

Joint Continuous-Discrete Model

The purpose of **nmm** is to allow the estimation of continuous-discrete model. Here continuous model is a system of nonlinear equations and the discrete model is logit (arising from various choice situations). By using quantile transformation, Lee (1983) was able to bind these two blocks together. Assuming that u is a vector of errors terms from systems of nonlinear regression and ϵ from discrete choice model, the framework developed in Munizaga et al. (2008) and introduced in Lee (1983) can be summarized as:

$$\underbrace{u = (u_1 u_2 \ldots u_g)' \sim \mathcal{N}(0, \Sigma)}_{\text{continuous block}} \qquad \Bigg| \qquad \underbrace{V_q \geq max_{m \neq q}\left\{U_m\right\} - \epsilon_q = \omega_q \sim Logistic}_{\text{discrete block}}$$

$$\Downarrow$$

$$y_q = \underbrace{\Phi^{-1}(F(V_q)) \geq \Phi^{-1}(\omega_q) = \omega_q^* \sim \mathcal{N}(0,1)}_{\text{Quantile transformation}}$$

$$\searrow \qquad \swarrow$$

$$\begin{gathered}
\text{Joint density} \\
f(u, \omega_q^* \leq y_q) = f(u) f(\omega_q^* \leq y_q | u) = f(u) \Phi(y_q | u) \\
\Downarrow \\
\text{Conditional moments:} E\left[y_q | u\right], Cov\left[y_q | u\right] \\
\Downarrow \\
\text{Theorem "Conditional normal distribution"} \\
\Downarrow \\
E\left[y_q | u\right] = \mu_{y_q} + \Sigma_{y_q u} \Sigma_{uu}^{-1} (u - \mu_u) \\
Cov\left[y_q | u\right] = \Sigma_{y_q y_q | u} = \Sigma_{y_q y_q} - \Sigma_{y_q u} \Sigma_{uu}^{-1} \Sigma_{y_q u}
\end{gathered}$$

As mentioned before, further in this paper we will refer to the system of nonlinear regressions as the continuous block and to logit as the discrete block. Then talking about correlations, we will differentiate between the intra and inter-block correlations. The first ones refer to the correlations between the error terms from the continuous block. The second ones refer to the correlations between continuous and discrete blocks. There is no intra-block correlation in the discrete block, as logit is based on the independence of irrelevant alternatives (IIA) assumption.

Assuming that one has three continuous equations, Q - alternatives and repeated observations can occur in the discrete part. The joint partitioned log-likelihood has the following form:

$$\begin{aligned}
LL = \sum_{i}^{J} \sum_{k \in A_i} \sum_{q}^{Q} \alpha_{ikq} \delta_{ikq} W_i \, log \Bigg(& \left(\phi(\eta_{1i})^{W_{1i}} \phi \left(\frac{\eta_{2i} - \mu_{\eta_{2i}|\eta_{1i}}}{\sigma_{\eta_2|\eta_1}} \right)^{W_{2i}} \phi \left(\frac{\eta_{3i} - \mu_{\eta_{3i}|\eta_{1i}\eta_{2i}}}{\sigma_{\eta_3|\eta_1\eta_2}} \right)^{W_{3i}} \right)^{WC_i} \\
& \Phi \left(\frac{y_{ikq} - \mu_{y_{ikq}|\eta_{1i}\eta_{2i}\eta_{3i}}}{\sigma_{y_q|\eta_1\eta_2\eta_3}} \right)^{W_{iq}} \Bigg)
\end{aligned} \tag{13}$$

Here i indicates a person, k is choice index and q - discrete alternative. α_{ikq} - is equal to 1, if alternative q is available at choice k of person i, and 0 otherwise. δ_{ikq} - is equal to 1, if alternative q is chosen at choice k of person i, and 0 otherwise. $\phi(.)$ and $\Phi(.)$ correspond to the density and distribution functions of the standard normal distribution. η_{mi} is the error term from the m^{th} continuous equation for individual i. $\mu_{y|x}$ and $\sigma_{y|x}$ denote the conditional mean and the standard deviation respectively. y_{ikq} is the quantile transformed probability of alternative q chosen by person i at choice k.

Additionally, W_{1i}, W_{2i}, W_{3i} are weights applied to the separate continuous equations. In some situation, this might be used to balance the log-likelihood. For example, for one observation of continuous block multiple discrete choices are available. Each individual continuous observation has to be cloned/repeated to match the number of discrete choice situations. The weights can be chosen to be proportional to the number of choices (n_i) made by each individual i. Other individual-specific weights that might be applied are: W_{iq} - for different discrete alternatives; WC_i - for all continuous equations (in `nmm` function argument `npaths_cont=TRUE`); W_i - for the whole system (n `nmm` function argument `npaths=TRUE`).

Usage of nmm Package

We start the introduction of **nmm** package with a simple mock-up example without a theoretical background. The joint continuous-discrete model (JCDM) introduced in Section "Joint Continuous-Discrete Model" and separate parts of it are estimated with `dataM` data set from **nmm** package. This dataset was created using `MathPlacement` object from **Stat2Data** (Cannon et al., 2019) package. It consists of results from the math placement exam at a liberal arts college.

```
install.packages("nmm")

library(nmm)
data("dataM", package="nmm")

#> # A tibble: 6 x 11
#>   Student Gender PSATM  SATM  ACTM  Rank  Size GPAadj PlcmtScore Recommends
#>     <int>  <int> <int> <int> <int> <int> <int>  <int>      <int> <fct>
#> 1       1      0    56    56    25     1   420     40         23 R0
#> 2       2      0    57    60    23     1    85     40         21 R0
#> 3       3      1    54    62    27    42   421     38         20 R0
#> 4       4      0    53    56    27     6    75     38         20 R0
#> 5       5      1    57    64    31    72   462     35         19 R0
#> 6       6      0    63    68    21    96   518     34         18 R0
#> # ... with 1 more variable: TooHigh <int>
```

Results of the placement exam can be found in column `PlcmtScore`. Based on these scores, a course was recommended (column - `Recommends`) to a student. Not all of the students took the recommended course. This can be seen from variables: `TooHigh`, `TooLow`, `RecTaken`. New column `DR_Course` was created to show what type of course was selected based on the available recommendation:

- `alow` - lower,
- `bnormal` - recommended,
- `chigh` - higher.

Number of students that chose specific course:

alow	bnormal	chigh
54	855	1534

More than a half of students took a higher course than the recommended one. In this simple example we will try to estimate a NMM consisting of:

- a system of nonlinear regressions (SNR) that models the score of math placement exam (`PlcmtScore`) as well as ACT Score in Math (`ACTM`) and
- a multinomial logit, which predicts the level of course taken: low, normal or high.

ML estimation is performed with `nmm` function. User has at least to supply four arguments:

- data as `data.frame`;
- equation type: for SNR use - `eq_type = "cont"`, for logit - `eq_type = "disc"`, for joint - `eq_type ="joint"`;
- formula as a character string/vector (SNR - `eq_c`, logit - `eq_d`, joint - supply both);
- parameter names (SNR - `par_c`, logit - `par_d`, joint - supply both).

The exported methods (`summary`, `logLik`, `AIC`, `AICc`, `BIC`, `hessian`) as well as internal methods (`gradient`, `bread`, `meat`) are partially based on **maxLik** package. They mostly help to calculate the different variants of covariance matrix (normal, robust and robust-clustered).

System of non-linear Regressions

We use a SNR, which is simple, but does not have a theoretical background:

$$PlcmtScore = e^{(a0+a1PSATM+a2Rank+a3Size)} + \epsilon_{PlcmtScore} \tag{14}$$
$$ACTM = e^{(c0+c1*GPAadj)} + \epsilon_{ACTM} \tag{15}$$

Here we try to estimate the imposed nonlinear relationship that describes math scores of placement exam (`PlcmtScore`) and ACT (`ACTM`). First, we formulate equations in a form of a character strings and define the parameters:

```
eq_c <- c("PlcmtScore ~ exp(a0 + a1 * PSATM + a2 * Rank  + a3 * Size) ",
          "ACTM ~ exp(c0 + c1 * GPAadj)")
par_c <- c(paste0("a", 0:3), paste0("c", 0:1))
```

In **nmm**, nonlinear multivariate regression model is estimated in the ML framework and provides a nice looking summary output of the estimated model. As in the most nonlinear estimation procedures, `nmm` uses starting values in the optimization. For continuous block, `get_start` function uses at first `nlsystemfit`, then `nmm` and `maxLik` functions. For discrete block, the `nls` function is used in combination with `nmm`. One can supply starting values to `nmm` function directly by using argument `start_v`. If not supplied, function `get_start` will be called automatically. For demonstration purpose, we do it explicitly here:

```
stv1 <- c(3.39, 0.001, -0.001, 0.001, 3.58, -0.001)
names(stv1) <- par_c
stv <- get_start(data = dataM, part = "cont", eq_c = eq_c, par_c = par_c, startvals = stv1)
```

Using `nmm` function and supplying: data (`data = dataM`), equations (`eq_c = eq_c`), their type (eq_type = "cont"), parameters (`par_c = par_c`) as well as starting values (`start_v = stv`), one performs the maximum likelihood estimation:

```
nmm_cont_res <- nmm(data = dataM, eq_type = "cont", start_v = stv, eq_c = eq_c,
                    par_c = par_c)

summary(nmm_cont_res)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 16 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -9477.365
#>   free parameters
#> Estimates:
#>      Estimate Std. error t value Pr(> t)
#> a0  3.3937870  0.0208091 163.092  <2e-16 ***
#> a1  0.0019029  0.0003318   5.735  <2e-16 ***
#> a2 -0.0012659  0.0000687 -18.420  <2e-16 ***
#> a3  0.0001869  0.0000174  10.730  <2e-16 ***
#> c0  3.5831095  0.0136083 263.303  <2e-16 ***
#> c1 -0.0077549  0.0003755 -20.653  <2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

`PSATM` score in Math and number of students in HS class(`Size`) have a positive impact on the score in the math placement exam (`PlcmtScore`), whereas the adjusted rank in HS class (`Rank`) has a negative one. Adjusted GPA (`GPAadj`) negatively affects ACT Score in Math.

The robust standard errors for β_{ML} can be calculated by setting argument `type` to `"robust"` in `summary`. Contemporaneous covariance matrix of error terms (Equation 5) is calculated with `nmm_sigma` function. If needed, $Var(\widehat{\Sigma}_{ML})$ can be estimated by maximizing the log-likelihood from Equation (4) with respect to Σ for the given $\widehat{\beta}_{ML}$.

```
nmm_sigma(nmm_cont_res)

#> $sd
#>      s_1      s_2
#> 8.358836 4.067014
#>
#> $correlation
#>           rho_1     rho_2
#> rho_1 1.0000000 0.8517698
#> rho_2 0.8517698 1.0000000
```

```
nmm_cont_sigma <- nmm_sigma(nmm_cont_res, estimate=TRUE)

summary(nmm_cont_sigma)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 9 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -9477.365
#> 3  free parameters
#> Estimates:
#>               Estimate Std. error t value Pr(> t)
#> sd_1          8.358834   0.120894   69.14  <2e-16 ***
#> sd_2          4.067011   0.058820   69.14  <2e-16 ***
#> corr(c_1&c_2) 0.851810   0.005585  152.53  <2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

Here *sd_i* is the standard deviation of error term *i* from the continuous block, *corr*(*c_i*&*c_j*) are intra-block correlations, *c_i* is an error term from the continuous block.

Setting parameter `best_method = TRUE` makes `nmm` apply all available optimization algorithms from package **maxLik** and search for the "best" result between them:

```
nmm_cont_best <- nmm(dataM, eq_c = eq_c, par_c = par_c, eq_type = "cont",
               start_v = stv, best_method = TRUE)
```

In this case, the "NR" algorithm performs the best, but the improvement over the "BFGS" is minimal:

```
cc <- 1000000
(logLik(nmm_cont_best)*cc - logLik(nmm_cont_res)*cc)/cc

#> [1] 0.0001512576
#> attr(,"LLv")
#>          res1      res2
#> [1,] -6408.307 -3069.058
```

One can choose a specific algorithm by setting `opt_method` argument (for all available methods, see documentation of `maxLik` function). One can also set `DEoptim_run_main = TRUE` to use the global optimization from **DEoptim** package.

Log-likelihood of each equation can be calculated separately with `logLik` function by supplying the argument `new_coef` or it is also saved as an attribute of the `nmm` object:

```
logLik(nmm_cont_res,
       new_coef = nmm_cont_res$estimate)

#> [1] -9477.365
#> attr(,"LLv")
#>          res1      res2
#> [1,] -6408.271 -3069.094
```

```
attributes(nmm_cont_best)$LL

#>          res1      res2
#> [1,] -6408.307 -3069.058
#> attr(,"names")
#> [1] "eq_1" "eq_2"
```

An object created with `nmm` inherits both classes of the `maxLik` object (`"maxLik"` and `"maxim"`):

```
class(nmm_cont_res)

#> [1] "nmm"    "maxLik" "maxim"
```

The `nmm` object has the following attributes:

```
attributes(nmm_cont_res) %>% names

#>  [1] "names"      "class"      "functions"  "data"       "LL"
#>  [6] "type"       "cont_e"     "eq_c"       "par_c"      "corr_joint"
```

```
attributes(nmm_cont_res)$functions %>% names

#> [1] "jfunc" "jgrad" "jhess"
```

Here:

- `names` - attributes from `maxLik`
- `class` - class of object (`"nmm"`, inherits `"maxLik"` and `"maxim"`)
- `functions` - functions used in the maximum likelihood estimation:
  - `jfunc` - the log-likelihood function,
  - `jgrad` - gradient function,

  - jhess - Hessian function.
  - if estimation type is "joint", other functions will appear here too.

- data - prepared data used in the estimation
- LL - log-likelihood per equation
- type - estimation type: "cont", "disc", "joint"
- cont_e - continuous equations used in the estimation with "par[]" instead of parameter names
- eq_c - continuous equations used in estimation with original parameter names
- par_c - parameter names
- corr_joint - if there is correlation between continuous and discrete blocks.

For the sake of comparison of the nmm estimation results with the ones from other packages, in2mm function was written. One can create nmm object with function in2nmm by supplying an appropriate nmm object (skeleton) and the new coefficients. All statistics are recalculated according to the new parameters/coefficients. **Order of the new coefficients is important**. It should match the order of coefficients in the output of nmm object. They are sorted according to the names.

First we estimate the same theoretical model with function nlsystemfit from **systemfit** (Henningsen and Hamann, 2007) package.

```
library(systemfit)

model <- lapply(eq_c, as.formula)
labels <- eq_c %>% gsub("\\~.*", "", .)
model.sur <- nlsystemfit("SUR", model, stv1, data = dataM,
                            eqnlabels = labels )
```

There is no "nice" summary output for "nlsystemfit.system" object. Thus, we create a table consisting of the estimated coefficients.

```
x <- c("b", "se", "t", "p")
coefs <- lapply(model.sur[["eq"]], function(y) Reduce(cbind, y[x])) %>% do.call(rbind, .)
coefs <- lapply(par_c, function(x)coefs[row.names(coefs)==x, ] %>% t %>%
         as.matrix %>% .[1,]) %>% do.call(rbind, .) %>%data.frame() %>%
         cbind(par_c, .)
colnames(coefs) <- c("Var", x)
coefs
```

```
#>   Var             b           se          t p
#> 1  a0  3.0938065735 0.0275195478 112.422144 0
#> 2  a1  0.0069750765 0.0004423654  15.767682 0
#> 3  a2 -0.0028691753 0.0000936795 -30.627570 0
#> 4  a3  0.0004198871 0.0000214210  19.601660 0
#> 5  c0  3.1349339795 0.0187170695 167.490641 0
#> 6  c1  0.0047152283 0.0005124161   9.201952 0
```

Next, we reorder values according to parameter names to match the order in the "skeleton" model:

```
ncoef <- coefs[order(coefs[, 1]), 2]
names(ncoef) <- sort(coefs[, 1])
nsem <- in2nmm(nmm_cont_res, new_coef = ncoef)
```

Now different goodness-of-fit measures can be computed for both models using diagnostics function from **nmm**:

```
diagnostics(nsem, which=c("Rx2adj"))
```

```
#>      stat    eq      value
#> 1:    Rx2 Total     0.4687
#> 2:    Rx2     1     0.4186
#> 3:    Rx2     2     0.0881
#> 4: Rx2adj Total     0.4681
#> 5: Rx2adj     1     0.4179
#> 6: Rx2adj     2     0.0877
#> 7:    AIC Total 19381.7863
#> 8:   AICc Total 19381.7518
#> 9:    BIC Total 19416.5922
```

```
diagnostics(nmm_cont_res, only_total = TRUE)
```

```
#>      stat    eq      value
#> 1:   RMSE Total     6.5758
#> 2:   MAPE Total    20.3969
#> 3:    Rx2 Total     0.2967
#> 4: Rx2adj Total     0.2960
#> 5:    AIC Total 18966.7298
#> 6:   AICc Total 18966.6953
#> 7:    BIC Total 19001.5357
```

```
sapply(model.sur[["eq"]],"[[", "adjr2")
```

```
#> [1] 0.4178554 0.0877303
```

Adjusted R^2 values from **systemfit** coincide with the ones calculated with `diagnostics`. According to information criteria (AIC, AICc, BIC), the coefficients found with **nmm** produce a better fit. Based on R^2, one would choose the model from `nlsystemfit`, but is not an appropriate measure of goodness-of-fit in this non-linear case.

Some other estimation arguments of `nmm` function that can be useful are discussed below. The additional parameter `numerical_deriv` tells `nmm` not to use the analytical versions of Hessian and gradient, but instead to employ `numericHessian` and `numericGradient` functions. This might be faster and quite reliable for some problems, but with more complex systems **nmm** advises to use the analytical counterparts and to keep argument `check_hess=TRUE`. This argument checks the invertibility of the Hessian to ensure the availability of standard errors in each optimization step. `maxLik` function does not check this with the final solution[2]. Thus, sometimes one can get estimates, but no standard errors. If one is not interested in them, one can ignore this.

By default, if an error (not invertible Hessian; infinity as a total value of log-likelihood; bad function evaluation with starting values) in optimization appears, `nmm` jumps to the next optimizer. Optimizers have the following order: "BFGS", "NR", "CG", "NM", "BHHH", "SANN". If they do not produce a "good" log-likelihood value, `nmm` uses for the last attempt the global optimization with `DEoptim`.

nmm presents several extensions/advancements of **systemfit**. First, non-linear system of equations can be estimated with ML. Secondly, global optimizer can be used by setting argument `DEoptim_run_main = TRUE`. Thirdly, `summary` provides a generalizing overview of the estimation. Fourthly, optimization can be performed using analytical gradient and Hessian functions. Finally, "robust" as well as "robust clustered" standard errors can be calculated.

logit estimation

To model the choice of chosen course, we estimate a simple logit with the following indirect utility function:

$$V_i = ASC_i + \beta_{SATM} SATM + \beta_{PlcmtScore} PlcmtScore$$

Here `SATM` and `PlcmtScore` are SAT and placement exam scores in Math, respectively. As before, one needs to define equations and parameters:

```
eq_d <- c("ASC1" ,
          "ASC2 + b1_2 * SATM + b2_2 * PlcmtScore",
          "ASC3 + b1_3 * SATM + b2_3 * PlcmtScore")

par_d <- c(paste0("ASC", 1:3),paste0("b", rep(1:2, rep(2,2)), "_", 2:3))
```

Parameter "ASC1" is not estimated, as the first alternative is always the base one. Before the model can be estimated with `nmm` function, data has to be prepared with the function `prepare_data`:

```
dataMp <- dataM%>% data.frame %>% prepare_data(. , choice="DR_Course", PeID = "Student")
```

Argument `choice` should get the name of the column with the chosen alternatives. In our case this is `DR_Course` column. `PeID` points to a column with individual/**Pe**rsonal-**ID**entification number. If it is unavailable, new column with name "PeID" is created.

```
dataMp %>% head(., 2)

#>   Gender PSATM SATM ACTM chc_1 chc_2 chc_3 choicen Course CourseSuccess GPAadj
#> 1      0    56   56   25     0     1     0       2    210             1     40
#> 2      0    57   60   23     0     1     0       2    117             1     40
#>   Grade npaths npaths_cont PeID PlcmtScore Rank Recommends RecTaken Size
#> 1     A      1           1    1         23    1         R0        1  420
#> 2     A      1           1    2         21    1         R0        1   85
#>   TooHigh TooLow WeID avl_2 avl_3 avl_1 wd_2 wd_3 wd_1
#> 1       0      0    1     1     1     1    1    1    1
#> 2       0      0    1     1     1     1    1    1    1
```

Function `prepare_data` creates additional columns that are needed for the logit or joint estimation:

- `WeID` - choice identifier, this is needed to allow for repeated observations

[2] Although usage of analytical derivatives is recommended, it is not a sufficient condition for invertibility of the Hessian, as choice of starting values is very important.

- `choisen` - shows which alternative was chosen
- chc_i - equal to 1, if alternative i is chosen and 0 otherwise, from Equation (13) - δ_{ikq}
- avl_i - equal to 1, if alternative i is available and 0 otherwise, from Equation (13) - α_{ikq}
- wd_i - weight applied to discrete choice equation i, from Equation (13) - W_{iq}
- `npaths_cont` - same weight applied to all continuous equations, from Equation (13) - $WC_i = 1/npaths\_cont$
- `npaths` - same weight applied to the whole system of equations, from Equation (13) - $W_i = 1/npaths$

Here we set argument `eq_type = "disc"` to reflect the equation type and supply equations (`eq_d`) as well as parameters (`par_d`) to the discrete part:

```
nmm_disc_res <- nmm(dataMp, eq_type = "disc", eq_d = eq_d, par_d = par_d)
```

Summary and goodness-of-fit measures of the estimated model:

```
summary(nmm_disc_res)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 12 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -1718.04
#>   free parameters
#> Estimates:
#>      Estimate Std. error t value  Pr(> t)
#> ASC2 -1.45176    1.62471  -0.894  0.37156
#> ASC3  3.04740    1.60152   1.903  0.05706 .
#> b1_2  0.14553    0.03314   4.391 1.13e-05 ***
#> b1_3  0.10210    0.03274   3.118  0.00182 **
#> b2_2 -0.13307    0.02367  -5.622  < 2e-16 ***
#> b2_3 -0.16820    0.02357  -7.136  < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

```
diagnostics(nmm_disc_res)

#>              stat    eq     value
#>  1:       MisClas Total   24.6691
#>  2:       MisClas     1    2.2104
#>  3:       MisClas     2   35.1617
#>  4:       MisClas     3   36.6353
#>  5: adjMcFadden Total    0.0508
#>  6: adjMcFadden     1    0.0899
#>  7: adjMcFadden     2    0.0345
#>  8: adjMcFadden     3    0.0404
#>  9:    McFadden Total    0.0546
#> 10:    McFadden     1    0.1239
#> 11:    McFadden     2    0.0423
#> 12:    McFadden     3    0.0502
#> 13:         AIC Total 3448.0794
#> 14:        AICc Total 3448.0449
#> 15:         BIC Total 3482.8853
```

Coefficients $b2\_2$ and $b2\_3$ are negative, indicating that the placement score (`PlcmtScore`) has a negative impact on the probability to choose a recommended course (`bnormal`) or a higher one (`chigh`). Whereas, the parameters of SATM indicate the positive impact. Logit is based on the independence of irrelevant alternatives (IIA) assumption, thus the variance-covariance matrix is not available for this type of estimation.

```
nmm_sigma(nmm_disc_res)
[1] "Variance-covariance matrix is not available for logit!"
```

Joint Estimation

The unique joint estimation is performed by setting `eq_type` to "joint" and supplying all of the equations as well as the parameters [3]:

```
nmm_joint_res <- nmm(dataMp, eq_type = "joint", eq_d = eq_d,
                     par_d = par_d,  eq_c = eq_c, par_c = par_c,
                     start_v = c(nmm_cont_res$estimate, nmm_disc_res$estimate))
```

```
summary(nmm_joint_res)
```

```
#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 336 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -11302.51
#>   free parameters
#> Estimates:
#>        Estimate Std. error t value  Pr(> t)
#> a0    3.3937727  0.0206137 164.637  < 2e-16 ***
#> a1    0.0014372  0.0003307   4.346 1.39e-05 ***
#> a2   -0.0014517  0.0000703 -20.649  < 2e-16 ***
#> a3    0.0001921  0.0000175  10.979  < 2e-16 ***
#> c0    3.5830792  0.0134672 266.059  < 2e-16 ***
#> c1   -0.0084227  0.0003732 -22.568  < 2e-16 ***
#> ASC2 -1.4517647  1.4366732  -1.011 0.312254
```

[3]In logit estimation, function `nmm` uses random sampling for the selection of starting values. Thus, results might slightly differ.

```
#> ASC3  3.0473990  1.4031310   2.172 0.029867 *
#> b1_2  0.1454940  0.0303067   4.801 1.60e-06 ***
#> b1_3  0.1021573  0.0296959   3.440 0.000581 ***
#> b2_2 -0.1329216  0.0212702  -6.249  < 2e-16 ***
#> b2_3 -0.1683413  0.0210008  -8.016  < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

For this data set and model, difference in the parameters is minimal. In other situation, the joint inference might be preferable as it might reflect the simultaneous decision making situation better. Here, first the score is recorded and only afterwards the course is recommended. This type of joint estimation is not available with other R packages and thus is the main innovation of the **nmm** package.

Expresions of Conditional Moments

Estimation of NMMs involves calculation of conditional moments for the normal distribution. **nmm** also provides a possibility to get TeX formulas of these expressions. Symbolic computation is performed with the help of Maxima (2014). This program is not called by the estimation function if number of equations is smaller than 5. To estimate a model with more than 4 equations (this happens automatically) or get TeX expressions, function `cond_expr` can be used. In such case user needs to install Maxima (2014) and include `maxima` into PATH system variable. Suppose one has 3 equations/variables and $u \sim \mathcal{N}(0, \Sigma)$ and we want to find $\mu_{\eta_3|\eta_1\eta_2}$ and $\Sigma_{\eta_3|\eta_1\eta_2}$:

```
neq <- 3
sdv <- rep(NA, neq)
mv <- rep(0, neq)
out <- cond_expr(neq, sdv, mv, tex = TRUE)
```

Variable `neq` defines number of equation, `sdv` is a vector of standard deviations and `mv` is vector of means. $\mu_{\eta_3|\eta_1\eta_2}$ is:

```
out$latex$mean
```

```
#> [1]
"$$-{{\\left(\\sigma_{1}\\,\\rho_{2,3}-\\sigma_{1}\\,\\rho_{1,2}\\,\\rho_{1,3}
\\right)\\,\\sigma_{3}\\,{\\it eps_2}+\\left(\\rho_{1,3}\\,\\sigma_{2}-\\rho_{
1,2}\\,\\sigma_{2}\\,\\rho_{2,3}\\right)\\,\\sigma_{3}\\,{\\it eps_1}}\\over{
\\left(\\sigma_{1}\\,\\rho_{1,2}^2-\\sigma_{1}\\right)\\,\\sigma_{2}}}$$"
```

$$-\frac{\left(\sigma_1\,\rho_{2,3}-\sigma_1\,\rho_{1,2}\,\rho_{1,3}\right)\,\sigma_3\,{\it eps_2}+\left(\rho_{1,3}\,\sigma_2-\rho_{1,2}\,\sigma_2\,\rho_{2,3}\right)\,\sigma_3\,{\it eps_1}}{\left(\sigma_1\,\rho_{1,2}^2-\sigma_1\right)\,\sigma_2}$$

$\Sigma_{\eta_3|\eta_1\eta_2}$ is:

```
out$latex$cov
```

```
#> [1]
"$${{\\left(\\rho_{2,3}^2-2\\,\\rho_{1,2}\\,\\rho_{1,3}\\,\\rho_{2,3}+\\rho_{1,
3}^2+\\rho_{1,2}^2-1\\right)\\,\\sigma_{3}^2}\\over{\\rho_{1,2}^2-1}}$$"
```

$$\frac{\left(\rho_{2,3}^2-2\,\rho_{1,2}\,\rho_{1,3}\,\rho_{2,3}+\rho_{1,3}^2+\rho_{1,2}^2-1\right)\,\sigma_3^2}{\rho_{1,2}^2-1}$$

Here eps_i correspond to the error vector u from equation i, σ_i - standard deviation, $\rho_{i,j}$ - correlation between u_i and u_j.

Estimation of Theoretical Model

The second part of demonstration focuses on the estimation of the joint time-use expenditure and travel mode choice model used in Munizaga et al. (2008). Its macroeconomic framework is introduced in next subsection. Jokubauskaitė et al. (2019) applied and estimated this model using the **nmm** package in combination with **RSGHB** (Dumont and Keller, 2019) to account for the observed panel structure. In Section "MAED Data", we introduce the unique Austrian data set (Aschauer et al. (2019)) used in that estimation. The theoretical time-use expenditure model (TUEM; continuous part) is presented and estimated in Subsection "Time-Use and Expenditure Model Estimation", discrete mode choice model (logit) - in Subsection "Discrete Model for Trip Selection" and the joint estimation of them - in Subsection "Joint Estimation".

Although, in this paper we present only two applications of **nmm**, it can be used in other situations there continuous and discrete choices are made by the same economic agent (for applications in energy and transportation field see Derakhshan et al. (2015)). Also, it can be used for the estimation of system of nonlinear regressions in maximum likelihood framework or to specify a more complex logit model.

Joint Time-Use Expenditure and Mode Choice Model

The joint time-use expenditure and mode choice model formulated in Munizaga et al. (2008) consists of two blocks. Both of them can be estimated separately or jointly as described in Section "Joint Continuous-Discrete Model". First block is a system of nonlinear regressions (continuous block) as presented in Jara-Díaz and Guevara (2003) and Jara-Díaz et al. (2008). The second one is the discrete travel mode choice model formulated as a logit (discrete block). In this section, we present the theoretical framework estimated in paper of Jokubauskaitė et al. (2019) and partly in Hössinger et al. (2020) (only continuous block). Both studies use the same data sets. Reduced versions of these data sets are also provided in the **nmm** package: `MAEDtimeExpenditure` and `MAEDtravel`.

We start with the continuous block - system of nonlinear regressions. In the framework developed in Jara-Díaz and Guevara (2003) and Jara-Díaz et al. (2008) individual maximizes her/his utility function:

$$U = T_w^{\theta_w} \prod_{i=1}^{n} T_i^{\theta_i} \prod_{j=1}^{m} E_j^{\phi_j} \tag{16}$$

Here utility U depends on: T_w - the amount of time assigned to work, T_i - the time assigned to activity i, and E_j - the expenditure assigned to good j. The exponents $\{\theta_w, \theta_i, \phi_j\}$ represent the elasticity of utility with respect to a specific input - work time, activity time, and expenditure good/group, respectively.

Individual faces budget and time constraints, as well as technical constraints on committed/necessary goods and activities:

$$wT_w + I - \sum_{j=1}^{m} E_j \geq 0 \ (\lambda) \qquad \text{(budget constraint)} \tag{17}$$

$$\tau - T_w - \sum_{i=1}^{n} T_i = 0 \ (\mu) \qquad \text{(time constraint)} \tag{18}$$

$$T_i - T_i^{Min} \geq 0 \ (\kappa_i) \qquad \text{(technical constraints on activities)} \tag{19}$$

$$E_j - E_j^{Min} \geq 0 \ (\eta_j) \qquad \text{(technical constraints on goods)} \tag{20}$$

Here w is the wage rate, I - income not related to work, τ - total available time (in our study it will be 168 hours). The technical constraints (Equation (19) and (20) on those committed activities and goods that are necessary for the personal and household maintenance (travel, rental cost, etc.), are not explicitly estimated, but inferred from the observations. The consumers are left with no other choice but to stick to the technical minimum (T_i^{Min},E_j^{Min}), which is data driven. Sums of such committed activities and goods are introduced into the time and budget constraints as T_c and E_c, respectively (Hössinger et al., 2020). $\{\lambda, \mu, \kappa_i, \eta_j\}$ are Lagrangian constants used in the solving of this maximization problem. The result is an optimal allocation of time and expenditure, given by the following system of equations:

$$T^*_{w12} = \frac{\left((\Phi+\theta_w)(\tau-T_c)+(\Theta+\theta_w)\frac{E_c}{w}\right)\pm\sqrt{D}}{2(\Theta+\Phi+\theta_w)} \tag{21}$$

$$\text{here } D = \left((\Phi+\theta_w)(\tau-T_c)+(\Theta+\theta_w)\frac{E_c}{w}\right)^2 -$$

$$-4(\Theta+\Phi+\theta_w)\theta_w(\tau-T_c)\frac{E_c}{w}$$

$$T^*_i = \frac{\theta_i}{\Theta}(\tau - T^*_w - T_c) \tag{22}$$

$$E^*_j = \frac{\phi_j}{\Phi}(wT^*_w - E_c) \tag{23}$$

Here $\Theta = \sum_{i\in A_f}\theta_i$, $\Phi = \sum_{j\in G_f}\phi_j$ with A_f and G_f being the index sets of freely chosen activities/goods and their complement sets A^c_f and G^c_f. $T_c = \sum_{i\in A^c_f} T^{min}_i$ and $E_c = \sum_{j\in G^c_f} E^{min}_j$ correspond to the total committed time and expenditures. One can reformulate each equation as $Y = x(\beta)+\epsilon$, where $\epsilon \sim N(0,\sigma)$ and x is a nonlinear function of parameter vector $\beta = \{\Phi, \theta_w, \Theta, \theta_i, \phi_i\}$.

The system of equations can be estimated with methods either from Subsection "Systems of Nonlinear Regressions" or from "Nonlinear Simultaneous Equation Model". In this paper we choose to estimate this model as a system of nonlinear regressions, substituting T_w expression into Equation (22) and (23). Also, parameters are normalized with respect to Θ. In the simplest case (two groups of freely chosen activities and goods), 4 parameters ($\{\theta_w, \theta_1, \phi_1, \Phi\}$) are estimated and 2 are calculated as the outcome ($\theta_2 = 1-\theta_1$, $\phi_2 = \Phi - \phi_1$). Actually, one could only estimate the labor supply given by Equation (21), but as Jokubauskaitė et al. (2019) has shown, the omission of additional information, might result in the estimation bias.

Another part of the joint time-use expenditure and discrete mode choice model is logit. This discrete part can have various specifications. If handled alone, the estimation would follow the procedure as described in Section "Multinomial logit". If both blocks should be estimated jointly, the estimation follows the procedure described in Section "Joint Continuous-Discrete Model".

MAED Data

The estimation of the joint time-use and discrete choice model requires a lot of information. Data needed for the continuous part could be retrieved from various Time-Use Surveys (TUS) and Consumer Expenditure Surveys (CES). Usually, both surveys gather information on different individuals and in order to estimate the TUM merging would need to be done. The same holds also for the discrete travel mode choice part, which could be estimated on the National Household Travel Survey (NHTS) data. To have a full information set needed for the joint estimation, one should combine TUS and CES and then with NHTS. Such synthesized data might help to get some insight, but will not represent the population very well. To tackle all the obstacles of such synthesis, Mobility-Activity-Expenditure Diary (MAED) was designed and presented by Aschauer et al. (2019). It was carried out in Austria in two waves: spring and autumn of 2015. Gathered information is in agreement with the reported data from the Austrian Time Use Survey (ATUS) and Austrian Consumer Expenditure Survey (ACES) (Aschauer et al. (2018)). The most important outcome of MAED is that it provides all the needed information (time-use, expenditure, travel mode choice) simultaneously for the same individual and no merging of the data sets is needed. It should be noted that only people with some kind of payed employment were observed. Hössinger et al. (2020) provides a very detailed summary to all the adjustments done to the data and Jokubauskaitė et al. (2019) presents a more detailed analysis on the travel mode choice.

The MAED data set has already been used in several studies to estimate various types of value of time in €/h. Using the mixed logit framework, Schmid et al. (2019) estimated the discrete choice part and got estimates for the Value of Travel Time Savings (VTTS) for four transport modes: walk, bike, public transport and car. This indicator shows the opportunity cost of travel. The continuous block was estimated in Hössinger et al. (2020). As an outcome, the Value of Leisure (VoL) and Value of Time Assigned to Work (VTAW) were calculated. The VoL represents the marginal utility of all freely chosen activities and VTAW - of work. The latter one is calculated as a difference between the VoL and the hourly wage rate. Taking the VTTS information from Schmid et al. (2019), Hössinger et al. (2020) has also calculated the Value of Time Assigned to Travel (VTAT=VoL-VTTS). This indicator represents the direct (dis)utility derived from travel time. Both parts were united in the joint estimation present in Jokubauskaitė et al. (2019) and standard errors for VTAT were reported. In Hössinger et al. (2020) this was not possible, as the estimates of VoL and VTTS came from two different frameworks.

MAED data allows to tackle and research many different problems. Jokubauskaitė et al. (2019)

Table 1: Aggregation into modeling variables

Activity	Var.	Activity	Var.	Expendirue	Var.	Expendirue	Var.
Work	T_w	Sleep	T_c	Leisure	E_{f1}	Insurance	E_c
Leisure	T_{f1}	Domestic	T_c	Accomm	E_{f1}	Other	E_c
Eating	T_{f2}	Personal	T_c	Electronic	E_{f1}	Service	E_c
Shopping	T_{f2}	Travel	T_c	Clothes	E_{f2}	Health	E_c
Unspec	T_{f2}	Education	T_c	Savings	E_{f3}	Furniture	E_c
		Other	T_c	Housing	E_c	Education	E_c
				Food	E_c	Financing	E_c
				Mobility	E_c		

noticed the significant difference between VoL for men and women. The main suspect for that was the time spent at work. According to MAED women spent 9 hr/week less in the paid work and 8.3 hr/week more in the unpaid work. This was put into test in Jokubauskaitė et al. (2020) by assigning value to domestic work based on market wage rate for substitute workers (Jokubauskaitė and Schneebaum, 2021). As a result, previously reported (Hössinger et al., 2020; Jokubauskaitė et al., 2019,) gender-specific VoL gap diminished.

In this paper we use only several variables needed for the basic JCDM estimation. Readers can contact co-author Reinhard Hössinger to get a full access to this rich and innovative data set. Time-Use and Expenditure data is saved in `MAEDtimeExpenditure` and the discrete travel mode choice data in `MAEDtravel` data sets.

Now we will start the presentation of the MAED data. Summary of the weekly (168 hours) time-use and expenditure data (`MAEDtimeExpenditure` object):

w	I	Tw	Tf1	Tf2	Tc	Td
12.13592	28.12353	37.83837	28.93532	11.44753	89.77092	14.2629

Ef1	Ef2	Ef3	Ec
79.99244	26.29829	40.02734	332.4444

Average wage rate is 12.14 EUR/h. People spend around 37.84 h/week at work and 14.26 at domestic work. Situation by gender:

w	I	Tw	Tf1	Tf2	Tc	Td
11.50962	28.96125	33.34791	28.08886	11.61352	94.94515	18.41886
12.76391	27.28353	42.34103	29.78408	11.28109	84.58264	10.09565

Ef1	Ef2	Ef3	Ec
66.48772	21.80035	36.29339	270.3211
93.53386	30.80845	43.77144	394.7365

One can clearly see that females work less in the official work (column T_w), but do more unofficial work (T_d). For information on column names see `?MAEDtimeExpenditure`, for aggregation of original data into the modeling variables see Table (1).

Now we look at the travel data `?MAEDtravel` needed for the logit estimation. Distribution of chosen transportation modes total and by gender:

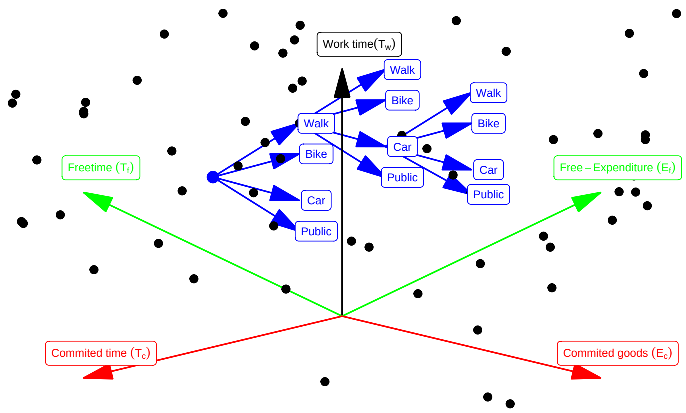
PeGenF	chc_1	chc_2	chc_3	chc_4
Total	2326	1036	11910	1855
female	1304	458	5998	921
male	1022	578	5912	934

Here indexes $\{1, 2, 3, 4\}$ correspond to the four travel modes $\{\text{walk, bike, car, public transport}\}$. Other than walking more and using bikes less often, gender-specific differences seem to be absent in the mode choice data. MAED is strongly dominated by car users, as 69.54 of total trips are done by car. This might not be a surprise, as this data set includes only workers.

It should be noted that each individual could make more than one trip. Summary of number of trips made by individuals is presented in the table below:

WeID	Min. : 4.00	1st Qu.:17.00	Median :23.00	Mean :23.24	3rd Qu.:28.00	Max. :56.00

In each given situation individual simultaneously chooses vector of time spent in work, on freely chosen/committed activities, expenditures as well as transport mode. This multidimensional decision making problem is portrayed in the simplified graphic below. Here each individual i has coordinates $(T_{wi}, T_{fi}, E_{fi}, T_{ci}, E_{ci}, Trip_1, \ldots, Trip_{n_i})$:



Time-Use and Expenditure Model Estimation

First step is to define equations and parameters that will be estimated:

```
eq_c <- c(
    "Tw ~ ((((PH) + (tw)) * (ta - Tc) + Ec/w * (1 + (tw)) +
    sqrt((Ec/w *(1 + (tw)) + (ta - Tc) * ((PH) + (tw)))^2 -
    4 * Ec/w * (ta -Tc) * (tw) * (1 + (PH) + (tw))))/(2 *
    (1 + (PH) + (tw))))",
  "Tf1 ~ (th1) * (ta - (Tw) - Tc)",
  "Ef1 ~ (ph1)/(PH) * (w*(Tw) - Ec) ",
  "Ef2 ~ (ph2)/(PH) * (w*(Tw) - Ec)")

eq_c[-1] %<>% gsub("Tw", eq_c[1] %>% gsub(".*~", "",.), .)
par_c <- c("tw", "PH", "th1", "ph1", "ph2")
```

Here we have 2 equations for freely chosen activities ($\{Tf1\}$, $Tf2$ is not estimated) and 3 for freely chosen goods ($\{Ef1, Ef2\}$, $Ef3$ is not estimated). Thus, 5 parameters are estimated ($\{tw, PH, th1, ph1, ph2\}$ - $\{\theta_w, \Phi, \theta_1, \phi_1, \phi_2\}$) and the rest can be calculated as: $\theta_2 = 1 - \theta_1$, $\phi_3 = \Phi - \phi_1 - \phi_2$. These equations are mathematical counterparts of Equation (21)-(23), where T_w expression is substituted into the rest of the equations.

```
r_cont <- nmm(data = MAEDtimeExpenditure, eq_c = eq_c, par_c = par_c,
              eq_type = "cont")


summary(r_cont)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 22 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -8820.291
#>   free parameters
#> Estimates:
#>      Estimate Std. error t value Pr(> t)
#> PH   0.417160   0.027616  15.106  <2e-16 ***
#> ph1  0.262488   0.017178  15.280  <2e-16 ***
#> ph2  0.085480   0.005866  14.571  <2e-16 ***
#> th1  0.733412   0.004206 174.381  <2e-16 ***
#> tw  -0.617428   0.088474  -6.979  <2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

Parameter `tw` corresponds to θ_w from the theoretical model. The negative sign shows that work has a negative marginal utility (MU). Positive values of `PH,ph1,ph2,th1` show that the MU from freely chosen activities and goods is positive. These parameter estimates can be further used to get monetary values of time: value of leisure, value of time assigned to work. For more details on this, please see Hössinger et al. (2020) and Jokubauskaitė et al. (2019).

Discrete Model for Trip Selection

Now we want to estimate the probability of choosing a specific transport mode. `MAEDtravel` has 4 alternatives: walk, bike, car, bus. For the estimation with `nmm`, ones needs to formulate the indirect utilities for each equation and to define parameter names:

```
eq_disc <- c("ASC1 + B11_dur * dur_1" ,
             "ASC2 + B12_dur * dur_2",
             "ASC3 + B13_dur * dur_3 + B20_cost * cost_3",
             "ASC4 + B14_dur * vdur_4 + B20_cost * cost_4")
par_d <- c(paste0("ASC", 1:4), paste0("B1", 1:4, "_dur"), "B20_cost")

r_disc <- nmm(MAEDtravel, eq_d = eq_disc, par_d = par_d,  eq_type = "disc",
              check_hess = FALSE, numerical_deriv = TRUE,
              weight_paths = FALSE)
```

New arguments:

- `check_hess` - if Hessian matrix should be checked for invertibility in each optimization step. If set to `TRUE`, estimation becomes very slow. Nevertheless, this is a good option to have for situations that are more complicated.
- `numerical_deriv` - if `TRUE`, uses numerical derivative instead of the analytical one.
- `weight_paths` - if `TRUE`, additional weights are applied to the repeated observations in the `MAEDtravel` data. Here it is set to `FALSE` as we do not want to penalize the log-likelihood based on the repeated observations.

```
summary(r_disc)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 11 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -8558.17
#>   free parameters
#> Estimates:
#>           Estimate Std. error t value  Pr(> t)
#> ASC2     -2.229819   0.073518 -30.330  < 2e-16 ***
#> ASC3     -0.236803   0.063698  -3.718 0.000201 ***
#> ASC4     -2.450802   0.078909 -31.059  < 2e-16 ***
#> B11_dur  -0.162260   0.003943 -41.155  < 2e-16 ***
#> B12_dur  -0.085414   0.002776 -30.773  < 2e-16 ***
#> B13_dur  -0.121708   0.005314 -22.901  < 2e-16 ***
#> B14_dur  -0.049332   0.003083 -16.000  < 2e-16 ***
#> B20_cost -0.794256   0.022630 -35.097  < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

Duration and cost of a trip has a negative effect on the probability of choosing a specific mode. The largest negative effect appears to be for walking (-0.16226). The parameters corresponding to the duration and cost can be further used to calculate value of travel time, which is an important indicator used in the cost-benefit analysis of transport related projects. For more details on this, please see Hössinger et al. (2020) and Jokubauskaitė et al. (2019). One can also have a non-linear indirect utility function in the logit estimation:

```
eq_disc1 <- c("ASC1 * 1 + B11_dur * dur_1 ** (H_3)" ,
             "ASC2 * 1 + B12_dur * dur_2",
             "ASC3 * 1 + B13_dur * dur_3 + B20_cost * cost_3",
             "ASC4 * 1 + B14_dur * vdur_4 + B20_cost * cost_4")

par_d1 <- c(paste0("ASC", 1:4), paste0("B1", 1:4, "_dur"), "B20_cost", "H_3")
```

```
r_disc1 <- nmm(MAEDtravel, eq_d = eq_disc1, par_d = par_d1,  eq_type = "disc",
              check_hess = FALSE, numerical_deriv = TRUE,
              weight_paths = FALSE)
```

Joint Estimation

To take into account the simultaneous choice situation, joint estimation is recommended. For this purpose, two data sets need to be merged.

```
jdata <- merge(MAEDtimeExpenditure, MAEDtravel)
```

Model estimation is now a combination from previous steps:

```
r_joint_BFGS <- nmm(jdata, eq_type = "joint",
                    eq_c = eq_c, par_c = par_c,
                    eq_d = eq_disc, par_d = par_d,
                    weight_paths_cont = TRUE,
                    weight_paths = FALSE, check_hess = FALSE)
```

Argument `weight_paths_cont` is set to `TRUE`, to apply weights according to the number of repeated observations in the continuous block. In our case this is needed, as `MAEDtimeExpenditure` data is artificially expanded to match the dimension of mode choice data. In other words, each individual has only one entry in the `MAEDtimeExpenditure` data, but multiple ones in the `MAEDtravel` data.

```
jdata %>% filter(PeID==100) %>%
  select("PeID", "WeID", "w", "Tw", "dur_1", "dur_2") %>% head(4)
```

```
#>   PeID  WeID     w Tw dur_1 dur_2
#> 1  100  6069 10.53 41   182  86.0
#> 2  100  6074 10.53 41   242 113.5
#> 3  100  6078 10.53 41   260 122.5
#> 4  100 12643 10.53 41   242 113.5
```

For the sake of comparison, the joint estimation is also performed without inclusion of the inter-block correlations.

```
r_no_corr_joint_BFGS <- nmm(jdata, eq_type = "joint", corrl = FALSE,
                    eq_c = eq_c, par_c = par_c,
                    eq_d = eq_disc, par_d = par_d,
                    weight_paths_cont = TRUE,
                    weight_paths = FALSE, check_hess = FALSE)
```

```
summary(r_no_corr_joint_BFGS)
```

```
#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 47 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -17379
#>   free parameters
#> Estimates:
#>          Estimate Std. error t value Pr(> t)
#> PH        0.41706    0.02822   14.78 < 2e-16 ***
#> ph1       0.26213    0.01754   14.95 < 2e-16 ***
#> ph2       0.08504    0.00597   14.26 < 2e-16 ***
#> th1       0.73367    0.00421  174.08 < 2e-16 ***
#> tw       -0.61753    0.09046   -6.83 < 2e-16 ***
#> ASC2     -2.22981    0.07186  -31.03 < 2e-16 ***
#> ASC3     -0.23680    0.06250   -3.79 0.00015 ***
#> ASC4     -2.45079    0.07713  -31.78 < 2e-16 ***
#> B11_dur  -0.16226    0.00387  -41.90 < 2e-16 ***
#> B12_dur  -0.08541    0.00277  -30.79 < 2e-16 ***
#> B13_dur  -0.12171    0.00532  -22.89 < 2e-16 ***
#> B14_dur  -0.04933    0.00308  -16.01 < 2e-16 ***
#> B20_cost -0.79426    0.02262  -35.11 < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

```
diagnostics(r_no_corr_joint_BFGS, only_total = TRUE)
```

```
#>            stat    eq      value
#>  1:        RMSE Total    22.6098
#>  2:        MAPE Total    51.5218
#>  3:         Rx2 Total     0.6008
#>  4:      Rx2adj Total     0.5994
#>  5:     MisClas Total     9.1201
#>  6: adjMcFadden Total     0.4646
#>  7:    McFadden Total     0.4651
#>  8:         AIC Total 34783.6536
#>  9:        AICc Total 34783.6324
#> 10:         BIC Total 34884.3830
```

```
summary(r_joint_BFGS)
```

```
#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 293 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -17148
#>   free parameters
#> Estimates:
#>          Estimate Std. error t value Pr(> t)
#> PH        0.43039    0.03335   12.91 < 2e-16 ***
#> ph1       0.20964    0.01654   12.67 < 2e-16 ***
#> ph2       0.08411    0.00684   12.29 < 2e-16 ***
#> th1       0.76543    0.00411  186.09 < 2e-16 ***
#> tw       -0.62156    0.10774   -5.77 < 2e-16 ***
#> ASC2     -2.23051    0.07156  -31.17 < 2e-16 ***
#> ASC3     -0.23868    0.06251   -3.82 0.00013 ***
#> ASC4     -2.44977    0.07718  -31.74 < 2e-16 ***
#> B11_dur  -0.15975    0.00382  -41.85 < 2e-16 ***
#> B12_dur  -0.08506    0.00279  -30.54 < 2e-16 ***
#> B13_dur  -0.12477    0.00537  -23.25 < 2e-16 ***
#> B14_dur  -0.05028    0.00312  -16.10 < 2e-16 ***
#> B20_cost -0.79227    0.02298  -34.48 < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

```
diagnostics(r_joint_BFGS, only_total = TRUE)
```

```
#>            stat    eq      value
#>  1:        RMSE Total    24.6409
#>  2:        MAPE Total    50.1988
#>  3:         Rx2 Total     0.5258
#>  4:      Rx2adj Total     0.5242
#>  5:     MisClas Total     9.1435
#>  6: adjMcFadden Total     0.4855
#>  7:    McFadden Total     0.4861
#>  8:         AIC Total 34321.5397
#>  9:        AICc Total 34321.5185
#> 10:         BIC Total 34422.2691
```

There is an improvement in the log-likelihood and the biggest change is observed in the estimation of parameters from the continuous block. Parameter `tw` becomes larger in absolute terms and thus the marginal disutility of work increases. The opposite can be said about the marginal utilities of freely chosen goods (`PH`), which increases. Discrete part parameters do not change much.

Most of the optimizers from **maxLik** package, with an exception of "SANN", perform local optimization. Therefore, the optimization sometimes might get stuck on a flat surface. Through our experience we have seen that this quite often happens with "BFGS" algorithm, which is used by default in `nmm`. To override this behavior, user can set argument `opt_method="NM"` or to a different value. Also, one can use previously created `nmm` object in the new estimation, to speed-up the creation of "`nmm`" skeleton (log-likelihood, gradient, Hessian functions, starting values). This is done by assigning the "`nmm`" object to argument `nmm_object = r_joint_BFGS`.

```
r_joint <- nmm(jdata, eq_type="joint",
               eq_c=eq_c, par_c=par_c,
               eq_d=eq_disc, par_d=par_d,
               weight_paths_cont = TRUE,
               weight_paths = FALSE, check_hess = FALSE,
               opt_method = "NM", miterlim = 1000000,
               nmm_object = r_joint_BFGS,
               start_v = r_joint_BFGS$estimate)
```

One could also use the evolutionary global optimization via the Differential Evolution algorithm (implemented in **DEoptim**). This would be done by setting argument '`DEoptim_run_main = TRUE`' and supplying `deconst`, which defines the upper and lower bounds of the parameters. By default, `DEoptim` is run with 1000 iterations. Afterwards, `maxLik` is called again with the starting values from `DEoptim`.

```
#  Caution: this procedure is quite slow!
r_joint_DE <- nmm(jdata, eq_type = "joint", eq_d = eq_disc, eq_c = eq_c,
                  par_c = par_c, par_d = par_d, weight_paths_cont = TRUE,
                  weight_paths = FALSE, check_hess = FALSE,
                  DEoptim_run_main = TRUE,
                  nmm_object = r_joint, start_v = r_joint$estimate)
```

```
summary(r_joint_BFGS)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> BFGS maximization, 293 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -17148
#>   free parameters
#> Estimates:
#>          Estimate Std. error t value Pr(> t)
#> PH        0.43039    0.03335   12.91 < 2e-16 ***
#> ph1       0.20964    0.01654   12.67 < 2e-16 ***
#> ph2       0.08411    0.00684   12.29 < 2e-16 ***
#> th1       0.76543    0.00411  186.09 < 2e-16 ***
#> tw       -0.62156    0.10774   -5.77 < 2e-16 ***
#> ASC2     -2.23051    0.07156  -31.17 < 2e-16 ***
#> ASC3     -0.23868    0.06251   -3.82 0.00013 ***
#> ASC4     -2.44977    0.07718  -31.74 < 2e-16 ***
#> B11_dur  -0.15975    0.00382  -41.85 < 2e-16 ***
#> B12_dur  -0.08506    0.00279  -30.54 < 2e-16 ***
#> B13_dur  -0.12477    0.00537  -23.25 < 2e-16 ***
#> B14_dur  -0.05028    0.00312  -16.10 < 2e-16 ***
#> B20_cost -0.79227    0.02298  -34.48 < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

```
summary(r_joint)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> Nelder-Mead maximization, 901 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -17146
#>   free parameters
#> Estimates:
#>          Estimate Std. error t value Pr(> t)
#> PH        0.39982    0.02966   13.48 < 2e-16 ***
#> ph1       0.19507    0.01480   13.18 < 2e-16 ***
#> ph2       0.07933    0.00628   12.64 < 2e-16 ***
#> th1       0.76513    0.00410  186.71 < 2e-16 ***
#> tw       -0.54371    0.09748   -5.58 < 2e-16 ***
#> ASC2     -2.25849    0.07173  -31.49 < 2e-16 ***
#> ASC3     -0.22831    0.06230   -3.66 0.00025 ***
#> ASC4     -2.41354    0.07688  -31.39 < 2e-16 ***
#> B11_dur  -0.15806    0.00379  -41.72 < 2e-16 ***
#> B12_dur  -0.08334    0.00276  -30.22 < 2e-16 ***
#> B13_dur  -0.12315    0.00535  -23.03 < 2e-16 ***
#> B14_dur  -0.04982    0.00311  -16.01 < 2e-16 ***
#> B20_cost -0.78856    0.02288  -34.46 < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

There is a slight improvement in the log-likelihood, but it is minimal. `r_joint` model mirrors a different world with lower marginal utility of freely chosen goods (`PH`) and a lower work disutility (`tw`). Now let's compare the results from `r_joint` and `r_joint_DE`:

```
summary(r_joint)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> Nelder-Mead maximization, 901 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -17146
#>   free parameters
#> Estimates:
#>          Estimate Std. error t value Pr(> t)
#> PH        0.39982    0.02966   13.48 < 2e-16 ***
#> ph1       0.19507    0.01480   13.18 < 2e-16 ***
#> ph2       0.07933    0.00628   12.64 < 2e-16 ***
#> th1       0.76513    0.00410  186.71 < 2e-16 ***
#> tw       -0.54371    0.09748   -5.58 < 2e-16 ***
#> ASC2     -2.25849    0.07173  -31.49 < 2e-16 ***
#> ASC3     -0.22831    0.06230   -3.66 0.00025 ***
#> ASC4     -2.41354    0.07688  -31.39 < 2e-16 ***
#> B11_dur  -0.15806    0.00379  -41.72 < 2e-16 ***
#> B12_dur  -0.08334    0.00276  -30.22 < 2e-16 ***
#> B13_dur  -0.12315    0.00535  -23.03 < 2e-16 ***
#> B14_dur  -0.04982    0.00311  -16.01 < 2e-16 ***
#> B20_cost -0.78856    0.02288  -34.46 < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

```
summary(r_joint_DE)

#> --------------------------------------------
#> Maximum Likelihood estimation
#> Nelder-Mead maximization, 375 iterations
#> Return code 0: successful convergence
#> Log-Likelihood: -17146
#>   free parameters
#> Estimates:
#>          Estimate Std. error t value Pr(> t)
#> PH        0.41022    0.03105   13.21 < 2e-16 ***
#> ph1       0.20056    0.01551   12.93 < 2e-16 ***
#> ph2       0.08161    0.00657   12.43 < 2e-16 ***
#> th1       0.76435    0.00410  186.43 < 2e-16 ***
#> tw       -0.57953    0.10250   -5.65 < 2e-16 ***
#> ASC2     -2.29009    0.07176  -31.91 < 2e-16 ***
#> ASC3     -0.23908    0.06226   -3.84 0.00012 ***
#> ASC4     -2.44211    0.07697  -31.73 < 2e-16 ***
#> B11_dur  -0.15841    0.00380  -41.64 < 2e-16 ***
#> B12_dur  -0.08163    0.00271  -30.09 < 2e-16 ***
#> B13_dur  -0.12175    0.00531  -22.93 < 2e-16 ***
#> B14_dur  -0.04818    0.00308  -15.64 < 2e-16 ***
#> B20_cost -0.78128    0.02270  -34.41 < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> --------------------------------------------
```

One again gets a different set of parameters, but the differences are minimal. Model without the inter-block correlations is the worst according to the Akaike's Information Criterion and the global optimization performed with **DEoptim** finds the best model.

Model	w/o corr, BFGS	w/ corr, BFGS	w/ corr, NM	w/ corr, DE
AIC	34783.65364	34321.53972	34318.91568	34317.86246

All these different optimization options might be helpful to push the optimizer from the local optimum. The new solution might not only be better with respect to the log-likelihood, but will generate a different set of parameters, which in their turn will result in different estimators of value of time.

Conclusions

Estimation of linear models is widely used and well established in the R environment. On the other hand, not many possibilities exist to estimate the nonlinear systems of equations. **nmm** package extends the already existing family of packages that estimate systems of nonlinear regressions (SNR, **systemfit**) and logit models (**mlogit**, **mnlogit**, **MNP**, **nnet**). What is more, it uniquely allows factoring in the correlations between these two types of systems by employing the framework proposed in Lee (1983).

For the robustness checks, estimation of separate blocks with `nmm` was tested and compared to **systemfit** and **mlogit** packages. This was done using `MathPlacement` data set from **Stat2Data**. All packages produce comparable results. No comparison of the joint estimation could be done, as other packages are not suitable for the joint estimation.

nmm provides several advancements over **systemfit** and **mlogit**. Firstly, it allows to calculate "robust" and "robust-clustered" standard errors. Secondly, estimation of system of nonlinear regressions (SNR) is extended to include the maximum likelihood estimation. Also, with **nmm** user can get a nice looking summary from the results of SNR estimation. Finally, models can be compared according to a wide range of goodness-of-fit measures (R^2, BIC, AIC, AICc) as well as to different types of pseudo R^2s (McFadden, adjusted McFadden, Cox&Snell, Nagelkerke). `nmm` also includes the usage of global optimization via the differential evolution algorithm (**DEoptim**). What is more, package **nmm** uniquely produces analytical versions of gradient and the Hessian functions, that can be used in the gradient based optimization. This might speed-up the optimization in some cases.

To sum up, the **nmm** package can be seen as an evolution of the nonlinear multivariate models estimation in the R environment. It allows a flexible design of logit and SNR, a unique modeling of correlations between continuous and discrete equations, and the perks of global optimization (**DEoptim**) in the ML framework.

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Simona Jokubauskaitė

Multi-Level Governance and Development ,
WU,
Austria

ORCiD: 0000-0002-4168-7393
simona.jokubauskaite@wu.ac.at

Friedrich Leisch

Institute of Statistics,
BOKU,
Austria

ORCiD: 0000-0001-7278-1983
Friedrich.Leisch@boku.ac.at

Reinhard Hössinger

Institute for Transport Studies,
BOKU,
Austria

ORCiD: 0000-0002-0090-0573
r.hoessinger@boku.ac.at

Package 'nmm'

January 7, 2021

Type Package

Title Nonlinear Multivariate Models

Version 0.9

Description Estimates a subset of nonlinear multivariate models (NMM):
system of nonlinear regressions (SNR), logit, and a joint model of SNR and logit.
'nmm' uniquely accounts for correlations between the error terms from nonlinear regressions
and the probabilities from logit models.
It also enables a very flexible design of logit: alternative-specific indirect utilities,
individual-specific choice set and number of actual choices.

Imports Rdpack, AER, mlogit, Hmisc, stats, gsubfn, abind, tidyr, plyr,
dplyr

Depends R (>= 4.0), systemfit, DEoptim, data.table, magrittr, maxLik

RdMacros Rdpack

License GPL (>= 2)

Encoding UTF-8

LazyData true

RoxygenNote 7.1.1

Suggests knitr, rmarkdown, recipes, Stat2Data

NeedsCompilation no

Author Simona Jokubauskaite [aut, cre],
Reinhard Hoessinger [aut],
Friedrich Leisch [aut]

Maintainer Simona Jokubauskaite <rteam.prog@gmail.com>

Repository CRAN

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R topics documented:

`addInter` *Add interactions*

Description

`addInter` add interactions into continuous equations.

Usage

```
addInter(eqcont, par_c, intv, inter_parl)
```

Arguments

eqcont	Vector of strings containing equations.
par_c	Names of coefficients.
intv	Vector of integers corresponding to coefficients to which interactions should be added.
inter_parl	Names of new coefficients (interactions).

Value

list: 1 - expressions of errors, equations, parameters to estimate

Examples

```
eq_c <- c("Tw ~ tw*w + ph1*Tc", "Tf1 ~ (1+w)^tw + ph1^3*Tc")
parl <- c("tw", "ph1")
intv <- c(1,0)
inter_parl <- c('yytw','yyph1')
res <- addInter(eq_c, parl, intv, inter_parl)
```

---

add_variable — add_variable *adds columns to the data matrix*

Description

add_variable adds columns to the data matrix

Usage

```
add_variable(data = data, dname = "chc", weights = NULL)
```

Arguments

data	data.frame, if dname=="chc" columns "chc_i" has to be in the data.
dname	if dname=="chc" (dummy for chosen alternative) dummy for the choice alternative added, if "weights" weights added
weights	Matrix with weights to be added to the data

Value

data.frame

Examples

```
chc <- c(1,2,1,4,3,1,4)
data <- data.frame(choice=chc, x=rnorm(length(chc)), y=rnorm(length(chc)))
add_variable(data, dname="chc")
ww <- c(1,1,1,2,2,2,3)
add_variable(data, dname="weights", weights=ww)
```

AICc *Adjusted Akaike's Information Criterion.*

Description

Calculates adjusted and Bayesian Information Criterion for nmm object

Usage

```
AICc(object, ..., k = 2)

## S3 method for class 'nmm'
AICc(object, ..., k = 2)

## Default S3 method:
AICc(object, ..., k = 2)

## S3 method for class 'nmm'
BIC(object, ..., k = 2)
```

Arguments

object	Fitted nmm model.
...	Not used.
k	Multiplication factor.

Value

a numeric value with the corresponding AIC, AICc, BIC.

Examples

```
library(systemfit)
data( ppine , package="systemfit")
hg.formula <- hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)
dg.formula <- dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)
labels <- list( "height.growth", "diameter.growth" )
model <- list( hg.formula, dg.formula )
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
model.sur <- nlsystemfit( "SUR", model, start.values, data=ppine, eqnlabels=labels )
eq_c <- as.character(c(hg.formula, dg.formula))
parl <- c(paste0("h", 0:3),paste0("d", 0:3))
res <- nmm(ppine, eq_c=eq_c, start_v=start.values, par_c=parl,
eq_type = "cont", best_method = FALSE)
aa <- in2nmm(res, model.sur$b)
AICc(res)
AICc(aa)
```

```
AIC(res)
AIC(aa)
BIC(res)
BIC(aa)
```

---

cond_expr *cond_expr returns moments of conditional multivariate normal distribution X|Y (last variable is dependent). Only expression for X|Y. Requires installation of Maxima software.*

Description

cond_expr returns moments of conditional multivariate normal distribution X|Y (last variable is dependent). Only expression for X|Y. Requires installation of Maxima software.

Usage

```
cond_expr(neq, sdv, mv, nconteq = neq - 1, tex = FALSE)
```

Arguments

neq	Number of equations/variables.
sdv	Vector of standard deviation of normally distributed variables, e.g. c(NA, NA, NA, 1) NA - unknown, any number - know.
mv	Vector of means of normally distributed variables, e.g. rep(0, 4).
nconteq	Number of continuous equations.
tex	i if TRUE TeX expressions from wxMaxima are returned.

Value

List of strings. First element is an expression of conditional mean and covariance. The second element is a TeX formula.

Examples

```
# this means that E[y3|y1,y2] and V[y3|y1,y2] will be returned
# all continuous w/ unknown means
## Not run:
# To run this, one needs to install Maxima software
res <- cond_expr(neq=3)
# 3 continuous w/ unknown means and the last one with mean 0 and sd 1, d|c1c2c3
res <- cond_expr(neq=4, sdv=c(NA, NA, NA, 1), mv=c(NA, NA, NA, 0))
# 2 continuous w/ unknown means and 2 discrete with mean 0 and sd 1, d1|c1c2c3d2
res <- cond_expr(neq=4, sdv=c(NA, NA,  1, 1), mv=c(NA, NA,  0, 0), nconteq=2)

## End(Not run)
```

cont_stats *Goodness of fit measures*

Description

Calculate RMSE, MAPE, R^2 and adjusted R^2

Usage

```
cont_stats(
  x,
  which = c("all", "RMSE", "MAPE", "Rx2", "Rx2adj"),
  only_total = FALSE
)
```

Arguments

x	Fitted nmm model.
which	What to calculate. Options: "all", "RMSE", "MAPE", "Rx2", "Rx2adj".
only_total	If TRUE, calculate statistics only for totals.

Value

matrix with Goodness of fit measures

Examples

```
library(systemfit)
data(ppine , package="systemfit")
hg.formula <- hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)
dg.formula <- dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)
labels <- list( "height.growth", "diameter.growth" )
model <- list( hg.formula, dg.formula )
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
model.sur <- nlsystemfit( "SUR", model, start.values, data=ppine, eqnlabels=labels )
eq_c <- as.character(c(hg.formula, dg.formula))
parl <- c(paste0("h", 0:3),paste0("d", 0:3))
res <- nmm(ppine, eq_c=eq_c, start_v=start.values, par_c=parl,
eq_type = "cont", best_method = FALSE)
cont_stats(res, which = "all")
```

convert_attr2exp — convert_attr2exp *converts symbolic attribute of derivative into expression object.*

Description

convert_attr2exp converts symbolic attribute of derivative into expression object.

Usage

```
convert_attr2exp(obj)
```

Arguments

obj	Symbolic expression of gradient or hessian

Value

combine expression of derivatives

Examples

```
eq1 <- parse(text="2*(log(sin(x)/log(x)))+x^4*log(x)+cos(y+x)")
tt1 <- deriv(eq1, c("x", "y"), hessian=TRUE)
r1 <- convert_attr2exp(extract_attr_deriv(tt1, "grad"))
r2 <- convert_attr2exp(extract_attr_deriv(tt1, "hessian"))
```

dat4cond_mean_cov_expr — *Log-likelihood expressions for cont. equations plus 1 discrete*

Description

Log-likelihood expressions for cont. equations plus 1 discrete

Usage

```
data(dat4cond_mean_cov_expr)
```

Format

An object of class list of length 4.

Examples

```
data(dat4cond_mean_cov_expr)
```

`dataM` *Example dataset*

Description

Data "MathPlacement" taken from Stat2Data package.

Usage

```
data(dataM)
```

Format

A data frame containing:

Student Identification number for each student

Gender 0=Female, 1=Male

PSATM PSAT score in Math

SATM SAT score in Math

ACTM ACT Score in Math

Rank Adjusted rank in HS class

Size Number of students in HS class

GPAadj Adjusted GPA

PlcmtScore Score on math placement exam

Recommends Recommended course: R0 R01 R1 R12 R2 R3 R4 R6 R8

Course Actual course taken

Grade Course grade

RecTaken 1=recommended course, 0=otherwise

TooHigh 1=took course above recommended, 0=otherwise

TooLow 1=took course below recommended, 0=otherwise

CourseSuccess 1=B or better grade, 0=grade below B

DR_Course according to recommendations, which level of course was taken: alow - lower, bnormal - recommended, chigh - higher

Details

Code for data modifications can be found in the example section.

Examples

```
data(dataM)
library(magrittr)
library(dplyr)
if (requireNamespace("recipes", quietly = TRUE)&requireNamespace("Stat2Data", quietly = TRUE)) {
data("MathPlacement", package="Stat2Data")
head(MathPlacement)
library(recipes)
# As some of the data is missing, k-nearest neighbors (knn) imputation is
# used to fill the gaps. This is done with recipes package and function
# step_knnimpute.
dataM <- recipe(~ ., data = MathPlacement) %>%
step_knnimpute(everything()) %>% prep() %>% juice()
# Afterwards we create a categorical variable that will show whether a
# student took a course which was too high, too low, the recommended one or
# something else happened:
dataM %<>% mutate(Student = 1:n(), DR_Course = case_when(
TooHigh == 1 ~ "chigh",
TooLow == 1 ~ "alow",
RecTaken == 1 ~ "bnormal",
TRUE ~"dother"
))
# We remove observations with ambiguous course status:
dataM %<>% filter(DR_Course!="dother")
dataM %>% select(DR_Course) %>% table %>% t
}
```

---

datmaxle *Log-likelihood expressions for cont. equations*

Description

Log-likelihood expressions for cont. equations

Usage

```
data(datmaxle)
```

Format

An object of class `list` of length 4.

Examples

```
data(datmaxle)
```

`datmlsem` *Log-likelihood expressions for cont. equations sem*

Description

Log-likelihood expressions for cont. equations sem

Usage

```
data(datmlsem)
```

Format

An object of class `list` of length 4.

Examples

```
data(datmlsem)
```

`diagnostics` *Goodness of fit measures for both parts*

Description

Calculation RMSE, misclassification and other goodness of fit measures.

Usage

```
diagnostics(
  x,
  xdigit = 4,
  which = "all",
  only_total = FALSE,
  cPseudoR = TRUE,
  cRs = TRUE
)
```

Arguments

`x`	Fitted `nmm` model.
`xdigit`	rounding number
`which`	What to calculate. Options: "all", "RMSE", "MAPE", "Rx2", "Rx2adj".
`only_total`	If `TRUE`, calculate statistics only for the whole system
`cPseudoR`	If `TRUE`, calculate pseudo R^2s.
`cRs`	Include "AIC", "AICc", "BIC"

Value

matrix with goodness of fit measures. `attribute corr` holds empirical variance-covariance matrix.

Examples

```
library(systemfit)
data( ppine , package="systemfit")
hg.formula <- hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)
dg.formula <- dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)
labels <- list( "height.growth", "diameter.growth" )
model <- list( hg.formula, dg.formula )
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
model.sur <- nlsystemfit( "SUR", model, start.values, data=ppine, eqnlabels=labels )
eq_c <- as.character(c(hg.formula, dg.formula))
parl <- c(paste0("h", 0:3),paste0("d", 0:3))
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
res <- nmm(ppine, eq_c=eq_c, start_v=start.values, par_c=parl, eq_type = "cont",
best_method = FALSE)
ressur <- in2nmm(res, new_coef=model.sur$b)
diagnostics(res)
diagnostics(ressur)

#example discrete
library(mlogit)
data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")
## a pure "conditional" model
mres <- summary(mlogit(mode ~ price + catch, data = Fish))
data <- prepare_data(Fish %>% data.frame %>% dplyr::select(-idx),
choice="alt", dummy="mode", PeID="chid", mode_spec_var = c("price", "catch"),
type="long")
eq_d <- c("a1 + p1 * price_1 + p2 * catch_2", "a2 + p1 * price_2 + p2 * catch_2",
             "a3 + p1 * price_3 + p2 * catch_3", "a4 + p1 * price_4 + p2 * catch_4")
par_d <- c(paste0("a", 1:4), paste0("p", 1:2))
res <- nmm(data, eq_d=eq_d, par_d=par_d,  eq_type="disc")
ncoef <- mres$coefficients
names(ncoef) <- par_d[-1]
resdisc <- in2nmm(res, new_coef = ncoef)
a <- diagnostics(res, xdigit=2)
a2 <- diagnostics(resdisc)
attributes(a2)$corr
```

`expr_ll_norm` *Log-likelihood expressions for cont. equations*

Description

Log-likelihood expressions for cont. equations

Usage

```
data(expr_ll_norm)
```

Format

An object of class `list` of length 8.

Examples

```
data(expr_ll_norm)
```

`expr_ll_norm_v2` *Another Log-likelihood expressions for cont. equations version 2*

Description

Another Log-likelihood expressions for cont. equations version 2

Usage

```
data(expr_ll_norm_v2)
```

Format

An object of class `list` of length 8.

Examples

```
data(expr_ll_norm_v2)
```

`extract_attr_deriv` *`extract_attr_deriv` converts attributes(hessian/gradient) of `deriv()` into a matrix of character strings.*

Description

`extract_attr_deriv` converts attributes(hessian/gradient) of `deriv()` into a matrix of character strings.

Usage

```
extract_attr_deriv(ex, attribute)
```

Arguments

`ex`	Expression of derivative. Results of `deriv()`.
`attribute`	"grad" for gradient or "hessian" for the Hessian matrix.

Value

Returns a matrix of character strings.

Examples

```
eq <- parse(text="2*(log(sin(x)/log(x)))+x^4*log(x)+cos(y+x)")
tt <- deriv(eq, c("x", "y"), hessian=TRUE)
g <- tt%>%extract_attr_deriv(., attribute = "grad")
h <- tt%>%extract_attr_deriv(., attribute = "hessian")
```

`formula2string`	`formula2string` *removes square brackets from the supplied expressions. Convert par[2] -> par2, sigma[2] -> sigma_2_, sigma[2,3] -> sigma_2x2*

Description

`formula2string` removes square brackets from the supplied expressions. Convert par[2] -> par2, sigma[2] -> sigma_2_, sigma[2,3] -> sigma_2x2

Usage

```
formula2string(x)
```

Arguments

x	String with square brackets.

Value

String without square brackets.

Examples

```
xx <- "par[1]*3+par[2]*par+rho1[1,2]+sigma1[2]"
formula2string(xx)
```

f_create *f_create creates functions for log-likelihood of different models.*

Description

f_create creates functions for log-likelihood of different models.

Usage

```
f_create(
  mn,
  data,
  fixed = 0,
  cheqs0 = NULL,
  separatenmm = FALSE,
  probt = NULL,
  tformula = NULL,
  hessian = NULL,
  transform = TRUE,
  sume = NULL
)
```

Arguments

mn	Expression, can be a list of equations.
data	Name of the data frame with which the function will be evaluated.
fixed	Integer, which parameter is fixed to be 0.
cheqs0	If continuous are supplied, include the expressions of errors.
separatenmm	if TRUE, separate log-likelihood for each equations is produced.
probt	Expressions of un-simplified probabilities with quantile transformation(qnorm(ifelse(P))).
tformula	unsimplified log(P)
hessian	Adds lines to check the Hessian, hessian should be the name of hessian function.
transform	if TRUE, adds lines to check conditional means
sume	Expression of summed likelihoods.

Value

Function.

Examples

```
eq_d <- c("ASC1 * 1 + B11_dur * dur_1" , "ASC2 * 1 + B12_dur * dur_2",
"ASC3 * 1 + B13_dur * dur_3 + B20_cost * cost_3 + B53_parkman * PbAvl_3",
"ASC4 * 1 + B14_dur * dur_4 + B20_cost * cost_4 + B34_serv * servIdx_4 + B44_stop * stopUs1R1_4")
parl <- c(paste0("ASC", 1:4), paste0("B1", 1:4, "_dur"), "B20_cost", "B53_parkman", "B34_serv",
"B44_stop")
obj <- get_par(parl, eq_d)
ffor <- obj$cheqs0
res <- MNlogitf(ffor, separatenmm=FALSE, transform=FALSE)
ff <- f_create(res$formula, data="data", fixed=1)
```

---

get_npar

`get_npar` *Get number of parameters or vector of parameters in supplied equations. Extracts the number of parameters used in equations. Parameters are given as par[1], ..., par[n].*

Description

`get_npar` Get number of parameters or vector of parameters in supplied equations. Extracts the number of parameters used in equations. Parameters are given as par[1], ..., par[n].

Usage

```
get_npar(x, values = FALSE)
```

Arguments

`x`	List of strings.
`values`	if `TRUE` returns the character vector with parameters (par[i]).

Value

Number of parameters or vector with parameters (in form par[i]).

Examples

```
eq_d <- c("ASC1 * 1 + B11_dur * dur_1" , "ASC2 * 1 + B12_dur * dur_2",
"ASC3 * 1 + B13_dur * dur_3 + B20_cost * cost_3 + B53_parkman * PbAvl_3",
"ASC4 * 1 + B14_dur * dur_4 + B20_cost * cost_4 + B34_serv * servIdx_4 + B44_stop * stopUs1R1_4")
parl <- c("ASC1", "B11_dur", "ASC2", "B12_dur", "ASC3", "B13_dur", "B20_cost", "B53_parkman",
"ASC4", "B14_dur", "B20_cost", "B34_serv", "B44_stop") %>% unique
disc_par <- get_par(parl, eq_d)
get_npar(disc_par$cheqs0)
get_npar(disc_par$cheqs0, values=TRUE)
```

get_par	get_par *replaces names of parameters with par[i].*

Description

get_par replaces names of parameters with par[i].

Usage

```
get_par(par, object)
```

Arguments

par	Names of parameters, vector of character strings.
object	List consisting formulas.

Value

A list object consisting of equations for errors with par[i] (cheqs0), the original list with formulas (eqlab), vector of parameters (parld, same as par), vector of parameters in form of par[i] (parn, exogenous variables (exog), endogenous variables (endog, in case of discrete ""), number of parameters in each equation (neq_par).

Examples

```
# System of Non-linear Regressions
eq_c <- c("hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)",
"dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)")
par_c <- c(paste0("h", 0:3),paste0("d", 0:3))
para_cont <- get_par(par_c, eq_c)
# Indirect utility functions for discrete choice:
eq_d <- c("a1 + p1 * price_1 + p2 * catch_2", "a2 + p1 * price_2 + p2 * catch_2",
"a3 + p1 * price_3 + p2 * catch_3", "a4 + p1 * price_4 + p2 * catch_4")
par_d <- c(paste0("a", 1:4), paste0("p", 1:2))
disc_par <- get_par(par_d, eq_d)
```

get_start	get_start *get starting values for discrete or continuous choice model.*

Description

get_start get starting values for discrete or continuous choice model.

Usage

```
get_start(
  eq_c = NULL,
  eq_d = NULL,
  data = NULL,
  part = "joint",
  datan = "data",
  fixed_term = FALSE,
  weight_paths = TRUE,
  weight_paths_cont = FALSE,
  data_weight = NULL,
  par_c = NULL,
  par_d = NULL,
  best_method = FALSE,
  startvals = NULL,
  DEoptim_run = FALSE,
  hessian = NULL,
  transform = TRUE,
  MNtypef = "logit",
  pardogit = NULL,
  opt_method = "BFGS",
  numerical_deriv = FALSE
)
```

Arguments

`eq_c`	Continuous equations errors.
`eq_d`	Discrete equations.
`data`	`data.frame` is used in the optimization.
`part`	Type of estimation: "joint", "cont", "disc".
`datan`	Name of `data.frame` used in the optimization.
`fixed_term`	if `TRUE`, includes fixed term in log-likelihood.
`weight_paths`	if `TRUE`, weights paths of the whole system.
`weight_paths_cont`	if `TRUE`, weight paths only in continuous part.
`data_weight`	`data.frame` with weights for continuous and discrete equations, same dim as `data`.
`par_c`	Names of parameters in continuous equations.
`par_d`	Names of parameters in discrete equations.
`best_method`	if `TRUE`, try all possible optimization methods and choose the one with the smallest likelihood.
`startvals`	Starting values, can be also `NULL`.
`DEoptim_run`	if `TRUE`, runs DEoptim for the optimization.
`hessian`	Name of hessian function.

transform	if TRUE, quantile transformation is applied.
MNtypef	"dogit" or "logit".
pardogit	"dogit" parameters.
opt_method	optimization method to use.
numerical_deriv	if TRUE, numerical derivatives are calculated in nmm function

Value

Starting values for discrete or continuous blocks.

Examples

```
# Example of discrete choice model
data("TravelMode", package = "AER")
eq_d <- c("ASC1 * 1 + B2_t * travel_1 + B3_v * vcost_1" ,
          "ASC2 * 1  + B2_t * travel_2 + B3_v * vcost_2",
          "ASC3 * 1  + B2_t * travel_3 + B3_v * vcost_3",
          "ASC4 * 1  + B2_t * travel_4 + B3_v * vcost_4")
parl <- c(paste0("ASC", 1:4), "B2_t", "B3_v")
obj <- get_par(parl, eq_d)

mode_spec_var <- c("wait", "vcost", "travel", "gcost")
data <- TravelMode
data$wait <- as.numeric(data$wait)
data[data$wait==0,"wait"] <- 0.000001 # add a small number to 0
data$travel <- as.numeric(data$travel)
data[data$travel==0,"travel"] <- 0.000001
data$vcost <- as.numeric(data$vcost)
data[data$vcost==0,"vcost"] <- 0.000001
data <- prepare_data(data, choice="mode", dummy="choice", PeID="individual", WeID="",
type="long", mode_spec_var =mode_spec_var, wc=FALSE)
stv <- get_start(eq_d=eq_d, data=data, datan="data", part="disc", par_d = parl,
transform = FALSE)

#example, system of equations
data("CreditCard", package="AER")
cdat <- CreditCard
cdat$income2 <- cdat$income^2
cdat$d_selfemp <- as.numeric(cdat$selfemp)
eq_c <- c("expenditure ~  b1*age + b2*income + b3*income2",
"income ~ a1*age + a2*d_selfemp + a3*dependents + a4*majorcards")
parl <- c(paste0("b", 1:3), paste0("a", 1:4))
para_cont <- get_par(parl, eq_c)
cheqs0 <- para_cont$cheqs0

stv <- get_start(eq_c = eq_c, data=cdat, datan="cdat", part="cont", par_c=parl)
```

grad_hess_eval — grad_hess_eval *forms function of gradient and Hessian of log-likelihood produced by* f_create.

Description

grad_hess_eval forms function of gradient and Hessian of log-likelihood produced by f_create.

Usage

```
grad_hess_eval(mn, parnl, hessian = FALSE, fixed = 0, data = "", cheqs0 = NULL)
```

Arguments

mn	Expression, can be a list of equations.
parnl	Names of parameters.
hessian	if TRUE, returns hessian function, otherwise gradient.
fixed	Integer, which parameter is fixed to be 0.
data	Name of the data frame with which the function will be evaluated.
cheqs0	If continuous are supplied, include the expressions of errors.

Value

A function for evaluation of gradient or Hessian.

Examples

```
eq_d <- c("ASC1 * 1 + B11_dur * dur_1" , "ASC2 * 1 + B12_dur * dur_2",
"ASC3 * 1 + B13_dur * dur_3 + B20_cost * cost_3 + B53_parkman * PbAvl_3",
"ASC4 * 1 + B14_dur * dur_4 + B20_cost * cost_4 + B34_serv * servIdx_4 + B44_stop * stopUs1R1_4")
parl <- c(paste0("ASC", 1:4), paste0("B1", 1:4, "_dur"), "B20_cost", "B53_parkman", "B34_serv",
 "B44_stop")
disc_par <- get_par(parl, eq_d)
ffor <- disc_par$cheqs0
parld <- disc_par$parld
res <- MNlogitf(ffor, separatenmm=FALSE, transform=FALSE)
parnl <- paste0("par", 1:length(parld))
gf <- grad_hess_eval (res, parnl, data="data", fixed=1)
hf <- grad_hess_eval (res, parnl, data="data", fixed=1, hessian=TRUE)
```

in2nmm — in2nmm *convert some estimation results into* nmm *object.*

Description

in2nmm convert some estimation results into nmm object.

Usage

```
in2nmm(to, new_coef)
```

Arguments

to	nmm object.
new_coef	New coefficients.

Value

nmm object.

Examples

```
#example  continuous nonlinear
library(systemfit)
data( ppine , package="systemfit")
hg.formula <- hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)
dg.formula <- dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)
labels <- list( "height.growth", "diameter.growth" )
model <- list( hg.formula, dg.formula )
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
model.sur <- nlsystemfit( "SUR", model, start.values, data=ppine, eqnlabels=labels )
eq_c <- as.character(c(hg.formula, dg.formula))
parl <- c(paste0("h", 0:3),paste0("d", 0:3))
res <- nmm(ppine, eq_c=eq_c, start_v=start.values, par_c=parl, eq_type = "cont",
best_method = FALSE)
aa <- in2nmm(res, model.sur$b)
summary(res, new_coef=model.sur$b, type="robust")
summary(aa, type="robust")
summary(res, type="robust")
```

logLik.nmm *Log-likelihood(LL) with supplied coefficients.*

Description

Log-likelihood(LL) with supplied coefficients.

Usage

```
## S3 method for class 'nmm'
logLik(
  object,
  new_coef = NULL,
  separatenmm = FALSE,
  transform = FALSE,
  methodopt = "NA",
  ...
)
```

Arguments

object	Object of class nmm.
new_coef	"New" coefficients for which LL should be calculated.
separatenmm	if TRUE, returns separate LL for each equation.
transform	if TRUE, do quantile transformation (normal quantiles).
methodopt	"NA" means that automatic algorithm was used in maxLik, if equal to "BHHH" will return LL for each individual.
...	some methods for this generic function require additional arguments.

Value

Returns log-likelihood.

Examples

```
#example  continuous nonlinear
library(systemfit)
data( ppine , package="systemfit")
hg.formula <- hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)
dg.formula <- dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)
labels <- list( "height.growth", "diameter.growth" )
model <- list( hg.formula, dg.formula )
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
model.sur <- nlsystemfit( "SUR", model, start.values, data=ppine, eqnlabels=labels )
eq_c <- as.character(c(hg.formula, dg.formula))
parl <- c(paste0("h", 0:3),paste0("d", 0:3))
```

```
res <- nmm(ppine, eq_c=eq_c, start_v=start.values, par_c=parl, eq_type = "cont",
 best_method = FALSE)
logLik(res)
logLik(res, new_coef=res$estimate)
logLik(res, new_coef=model.sur$b)
#example discrete
library(mlogit)
data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")
## a pure "conditional" model
mres <- summary(mlogit(mode ~ price + catch, data = Fish))
data <- prepare_data(Fish %>% data.frame %>% dplyr::select(-idx),
choice="alt", dummy="mode", PeID="chid", mode_spec_var = c("price", "catch"),
type="long")
eq_d <- c("a1 + p1 * price_1 + p2 * catch_2", "a2 + p1 * price_2 + p2 * catch_2",
             "a3 + p1 * price_3 + p2 * catch_3", "a4 + p1 * price_4 + p2 * catch_4")
par_d <- c(paste0("a", 1:4), paste0("p", 1:2))
res <- nmm(data, eq_d=eq_d,  eq_type="disc", fixed_term=FALSE, par_d=par_d,
best_method=FALSE)
logLik(res)
logLik(res, new_coef=res$estimate)
logLik(res, new_coef=mres$coefficients)
```

MAEDtimeExpenditure *Time-use and expenditure dataset*

Description

Data gathered in Austria in 2015 according to Mobility-Activity-Expenditure-Dairy (MAED), which reported all trips, activities (time use) and expenditures of 737 persons over a whole week

Usage

```
data(MAEDtimeExpenditure)
```

Format

A data frame containing:

PeID individual index

PeGenF gender of the individual

PeAge age in years

PeEduc education level

PeEmploy employment state

HhCh type of household: with children or without children

w hourly wage rate, EUR/h

I income not realted to work, EUR/week

Tw time spent at work, h/week

Tf1 freely chosen activities group 1 (leisure), h/week

Tf2 freely chosen activities group 2 (eating, shopping, unspecified), h/week

Tc time spent on committed activities (sleep, domestic work, personal care, travel, education, other), h/week

Ef1 freely chosen expenditure group 1 (leisure, accommodation, electronics), EUR/week

Ef2 freely chosen expenditure group 2 (clothes), EUR/week

Ef3 freely chosen expenditure group 3 (savings), EUR/week

Ec committed expenditures (housing, food, mobility, insurance, other, services, health, furniture, education, financing), EUR/week

ta total time budget = 168 h/week

Td time spent on domestic chores, h/week. Td is part of Tc.

Details

Time and expenditure data correspond to weekly totals. Time in hours and expenditure in EUR.

For more on data collection and description see (Aschauer et al. 2018) and (Aschauer et al. 2019).

A variant of this dataset was used in: (Schmid et al. 2019),(Jokubauskaite et al. 2019) and (Hoessinger et al. 2020).

To get the full dataset please contact r.hoessinger@boku.ac.at.

References

Aschauer F, Roesel I, Hoessinger R, Kreis BH, Gerike R (2019). "Time use, mobility, expenditure: An innovative survey design for understanding individual trade-off processes." *Transportation*, **46**, 307–339. doi: 10.1007/s1111601899619.

Aschauer F, Hoessinger R, Axhausen KW, Schmid B, Gerike R (2018). "Implications of survey methods on travel and non-travel activities. A comparison of the Austrian national travel survey and an innovative mobility-activity-expenditure diary (MAED)." *European Journal of Transport and Infrastructure Research*, **18**, 4–35. doi: 10.3929/ethzb000181072.

Hoessinger R, Aschauer F, Jara-Diaz S, Jokubauskaite S, Schmid B, Peer S, Axhausen KW, Gerike R (2020). "A joint time-assignment and expenditure-allocation model: value of leisure and value of time assigned to travel for specific population segments." *Transportation*, **47**, 1439–1475. doi: 10.1007/s1111601910022w.

Jokubauskaite S, Hoessinger R, Aschauer F, Gerike R, Jara-Diaz S, Peer S, Schmid B, Axhausen KW, Leisch F (2019). "Advanced continuous-discrete model for joint time-use expenditure and mode choice estimation." *Transportation Research Part BMethodological.*, **129**, 397–421. doi: 10.1016/j.trb.2019.09.010, `https://www.sciencedirect.com/science/article/pii/S0191261518308245`.

Schmid B, Jokubauskaite S, Aschauer F, Peer S, Hoessinger R, Gerike R, Jara-Diaz SR, Axhausen KW (2019). "A pooled RP/SP mode, route and destination choice model to investigate mode and user-type effects in the value of travel time savings." *Transportation Research Part APolicy and Practice*, **124**, 262–294. doi: 10.1016/j.tra.2019.03.001, `https://www.sciencedirect.com/science/article/pii/S0965856418301721`.

Examples

```
data(MAEDtimeExpenditure)
```

`MAEDtravel` *Trip dataset*

Description

Data gathered in Austria in 2015 according to Mobility-Activity-Expenditure-Dairy (MAED), which reported all trips, activities (time use) and expenditures of 737 persons over a whole week.

Usage

```
data(MAEDtravel)
```

Format

A dataframe containing:

PeID individual index

PeGenF gender of the individual

PeAge age in years

PeEduc education level

PeEmploy employment state

HhCh type of household: with children or without children

WeID trip index

choice chosen mode: 1 - walk, 2 - bike, 3 - car, 4 -public transport (PT)

dist trip distance, km

avl_1 availability dummy for mode 1, walk

avl_2 availability dummy for mode 2, bike

avl_3 availability dummy for mode 3, car

avl_4 availability dummy for mode 4, PT

chc_1 choice dummy for mode 1, walk

chc_2 choice dummy for mode 2, bike

chc_3 choice dummy for mode 3, car

chc_4 choice dummy for mode 4, PT

cost_3 cost of car mode

cost_4 cost of PT mode

dur_1 trip duration with mode 1, minutes

dur_2 trip duration with mode 2, minutes

dur_3 trip duration with mode 3, minutes

vdur_4 in vehicle time mode 4, minutes

acc_4 time to stop or from stop for mode 4, minutes

HhCarPark dummy, car parking at home available

JobCarPark dummy,car parking at workplace available

PbAvl_3 dummy, ar parking restrictions (time and/or cost) in-force at the destination of the trip

servIdx_4 public transport service interval in minutes

stopUs1R1_4 necessary number of changes to reach the destination with public transport

leis trip purpose leisure, effect coding

work trip purpose work, effect coding

oth trip purpose other, effect coding

int_1 inertia for mode 1

int_2 inertia for mode 2

int_3 inertia for mode 3

int_4 inertia for mode 4

Details

For more on data collection and description see (Aschauer et al. 2019) and (Aschauer et al. 2018).

A variant of this dataset was used in: (Schmid et al. 2019), (Jokubauskaite et al. 2019) and (Hoessinger et al. 2020).

To get the full dataset please contact r.hoessinger@boku.ac.at.

Transport modes available: walk, bike, car, public transport (PT). The inertia variable (int_i) is a dummy, which is equal to one if the mode chosen by a person for a trip at the start of the current tour is the same as the one chosen in the previous tour made for the same purpose, and zero otherwise. Variables for trip purpose (leis, work, oth) were created using the effect coding.

References

Aschauer F, Hoessinger R, Axhausen KW, Schmid B, Gerike R (2018). "Implications of survey methods on travel and non-travel activities. A comparison of the Austrian national travel survey and an innovative mobility-activity-expenditure diary (MAED)." *European Journal of Transport and Infrastructure Research*, **18**, 4–35. doi: 10.3929/ethzb000181072.

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Hoessinger R, Aschauer F, Jara-Diaz S, Jokubauskaite S, Schmid B, Peer S, Axhausen KW, Gerike R (2020). "A joint time-assignment and expenditure-allocation model: value of leisure and value of time assigned to travel for specific population segments." *Transportation*, **47**, 1439–1475. doi: 10.1007/s1111601910022w.

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Examples

```
data(MAEDtravel)
```

---

maxle — `maxle` *returns expression of log-likelihood (LL) of joint normal distribution.*

Description

`maxle` returns expression of log-likelihood (LL) of joint normal distribution.

Usage

```
maxle(cheqs0, fixed_term = TRUE)
```

Arguments

`cheqs0`	Strings defining equations of errors. Systems of Nonlinear Regressions (SNR) variant.
`fixed_term`	if `TRUE` fixed term -(k/2)*log(2*pi) (k number of equations) is included

Value

List with LL expressions of joint normal distribution, first element is string with expression for derivative calculations, the second - string for evaluation.

Examples

```
# normal distribution
eq_c <- c("Tw ~ ((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) -(1 + (PH))) +
sqrt((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 * (1 + (PH) +
 (tw)) *(-(PH) * (ta - Tc + 2) + (1 - (tw) * (ta - Tc + 2)) * (2/w -Ec/w))))/(2 * (1 + (PH) +
  (tw)))",
"Tf1 ~ (th1) * (ta - (((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) *(Ec/w - 2/w) - (1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta -Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *(1 + (PH) +
 (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 * (1 + (PH) +
  (tw)))) -Tc + 2) - 1",
"Ef1 ~ (ph1)/(PH) * (w * (((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) -
(1 + (PH))) + sqrt((((PH) + (tw)) *(ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 -4 *
 (1 + (PH) + (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 *
  (1 + (PH) + (tw)))) -Ec + 2) - 1")
parl <- c("tw","PH","th1","ph1")
```

```
para_cont <- get_par(parl, eq_c)
cheqs0 <- para_cont$cheqs0
res <- maxle(cheqs0=cheqs0)
```

maxle_p — maxle_p *returns expression of partitioned log-likelihood. f(y1,y2,..,yn)=f(y1)f(y2|y1)f(y3|y2y1)...f(yn|y1..y(n-1))*

Description

maxle_p returns expression of partitioned log-likelihood. f(y1,y2,..,yn)=f(y1)f(y2ly1)f(y3ly2y1)...f(ynly1..y(n-1))

Usage

```
maxle_p(cheqs0, fixed_term = TRUE, version2 = TRUE)
```

Arguments

cheqs0	Strings defining equations of errors.
fixed_term	if TRUE fixed term -(k/2)*log(2*pi) (k number of equations) is included
version2	another formulation of log-likelihood

Value

List. First element is expression of joint distribution for derivatives, second for evaluation, third latex, fourth marginal distributions for each variable.

Examples

```
# joint normal distribution
eq_c <- c("Tw ~ ((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) -(1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 * (1 + (PH) +
  (tw)) *(-(PH) * (ta - Tc + 2) + (1 - (tw) * (ta - Tc + 2)) * (2/w -Ec/w))))/(2 * (1 + (PH) +
    (tw)))",
"Tf1 ~ (th1) * (ta - (((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) *(Ec/w - 2/w) - (1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta -Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *(1 + (PH) +
  (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 * (1 + (PH) +
    (tw)))) -Tc + 2) - 1",
"Ef1 ~ (ph1)/(PH) * (w * (((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) - (1 +
 (PH))) + sqrt((((PH) + (tw)) *(ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 -4 *
  (1 + (PH) + (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 *
   (1 + (PH) + (tw)))) -Ec + 2) - 1")
parl <- c("tw","PH","th1","ph1")
para_cont <- get_par(parl, eq_c)
cheqs0 <- para_cont$cheqs0
res <- maxle_p(cheqs0=cheqs0)
```

`mlsem` *mlsem returns expression of log-likelihood for joint normal distribution, for maximum likelihood (ML), Simultaneous Equations Models (SEM) variant.*

Description

`mlsem` returns expression of log-likelihood for joint normal distribution, for maximum likelihood (ML), Simultaneous Equations Models (SEM) variant.

Usage

```
mlsem(cheqs0, fixed_term = TRUE)
```

Arguments

`cheqs0`	Strings defining equations of errors.
`fixed_term`	if `TRUE` fixed term -(k/2)*log(2*pi) (k number of equations) is included

Value

List with LL expressions of joint normal distribution, first element is string with expression for derivative calculations, the second - string for evaluation.

Examples

```
# normal distribution
eq_c <- c("Tw ~ ((((PH) + (tw)) * (ta - Tc) + Ec/w * (1 + (tw)) + sqrt((Ec/w *(1 + (tw)) +
(ta - Tc) * ((PH) + (tw)))^2 - 4 * Ec/w * (ta -Tc) * (tw) * (1 + (PH) + (tw))))/(2 * (1 + (PH)
 + (tw))))",
"Tf1 ~ (th1) *(Tw - Tc)",
"Ef1 ~ (ph1)/(PH) * (w * Tw - Ec)")
parl <- c("tw","PH","th1","ph1")
para_cont <- get_par(parl, eq_c)
cheqs0 <- para_cont$cheqs0
res <- mlsem(cheqs0, fixed_term=FALSE)
```

`MNlogitf` *`MNlogitf` or `MNdogitf` returns log-likelihood(LL) expression for discrete equations of "logit" or "dogit" model.*

Description

`MNlogitf` or `MNdogitf` returns log-likelihood(LL) expression for discrete equations of "logit" or "dogit" model.

Usage

```
MNlogitf(ffor, transform = FALSE, separatenmm = FALSE, weight_disc = FALSE)

MNdogitf(
  ffor,
  transform = FALSE,
  separatenmm = FALSE,
  weight_disc = FALSE,
  ppardogit = NULL
)
```

Arguments

ffor	Discrete choice equations.
transform	if TRUE, quantile transformation (normal) is applied.
separatenmm	if TRUE, equation specific LL is calculated
weight_disc	if TRUE, equations will include equation specific weights.
ppardogit	"dogit" parameters only used in MNdogitf

Value

formula	simplified LL for each equation or joint
probs	probability expression unsimplified
expr	LL string simplified
formulat	unsimplified log(P), only if transform is TRUE
probt	unsimplified P with quantile transformation qnorm(ifelse(P)), only if transform is TRUE
expreteso	unsimplified pnorm((qnorm(P)-mean)/sd), only if transform is TRUE
probte	qnorm(P)
tval	If quantile transformation is applied
sume	Denominator of logit probability

Functions

- `MNlogitf`: returns log-likelihood(LL) expression for discrete equations of "logit" model.
- `MNdogitf`: returns log-likelihood(LL) expression for discrete equations of "dogit" model.

Examples

```
eq_d <- c("ASC1 * 1 + B11_dur * dur_1" , "ASC2 * 1 + B12_dur * dur_2",
"ASC3 * 1 + B13_dur * dur_3 + B20_cost * cost_3 + B53_parkman * PbAvl_3",
"ASC4 * 1 + B14_dur * dur_4 + B20_cost * cost_4 + B34_serv * servIdx_4 + B44_stop * stopUs1R1_4")
parl <- c(paste0("ASC", 1:4), paste0("B1", 1:4, "_dur"), "B20_cost", "B53_parkman", "B34_serv",
"B44_stop")
disc_par <- get_par(parl, eq_d)
```

```
ffor <- disc_par$cheqs0
res_l <- MNlogitf(ffor, separatenmm=FALSE, transform=FALSE)
res_d <- MNdogitf(ffor, separatenmm=FALSE, transform=FALSE)
```

`nmm` *Maximum likelihood estimation of nonlinear multivariate models (NMM).*

Description

`nmm`, `nmm_sigma` and `summary` are the main functions used for the estimation of NMM.

- `nmm` - Maximum likelihood estimation of nonlinear multivariate models (NMM)
- `nmm_sigma` - Optimizes the covariance matrix
- `summary` - returns summary of `nmm` object with "normal", "robust" or "clustered" standard errors. With option `new_coef` one can supply new coefficients and test their significance.

Usage

```
nmm(
  data,
  eq_type = c("joint", "cont", "disc"),
  eq_d = NULL,
  eq_c = NULL,
  par_c = NULL,
  par_d = NULL,
  start_v = NULL,
  check_hess = TRUE,
  corrl = TRUE,
  weight_paths = TRUE,
  weight_paths_cont = FALSE,
  data_weight = NULL,
  estimate = TRUE,
  fixed_term = FALSE,
  best_method = FALSE,
  DEoptim_run = FALSE,
  hessian = "joint_hess",
  print_out = FALSE,
  diff_hessian = FALSE,
  bayesian_random = FALSE,
  DEoptim_run_main = FALSE,
  deconst = 2,
  numerical_deriv = FALSE,
  best_method4start = FALSE,
  eqsys = "sur",
  miterlim = 10000,
  opt_method = "BFGS",
```

```
  try_last_DEoptim = TRUE,
  transform = NULL,
  MNtypef = "logit",
  nmm_object = NULL
)

## S3 method for class 'nmm'
summary(object, type = "normal", new_coef = NULL, ...)

nmm_sigma(
  object,
  methodopt = "BFGS",
  try_1good = TRUE,
  try_DEoptim = FALSE,
  try_diff_method = FALSE,
  trace = FALSE,
  estimate = FALSE
)
```

Arguments

data	data.frame used in the optimization.
eq_type	Possible options "joint", "cont", "disc".
eq_d	Discrete equations.
eq_c	Continuous equations.
par_c	Parameters from continuous equations.
par_d	Parameters from discrete equations.
start_v	Starting values for optimization. If NULL, starting values are found by get_start function.
check_hess	If TRUE, check the Hessian.
corrl	If TRUE, correlation between blocks (continuous and discrete).
weight_paths	If TRUE, weight according to the number of choices made by individual i will be added to the whole system.
weight_paths_cont	If TRUE, if only to continuous part should be weighted.
data_weight	Data weight matrix.
estimate	If TRUE, estimation is performed.
fixed_term	If TRUE, includes fixed term to continuous equation block.
best_method	If TRUE, all optimizers are checked.
DEoptim_run	If TRUE, runs DEoptim in generation of starting values.
hessian	String, name of the Hessian function.
print_out	If TRUE, prints out log-likelihood for each equation.
diff_hessian	If TRUE, for changing hessian and gradient with weights.

bayesian_random
: If TRUE, than par[1] is changed to par[,1] to be used for optimization of random parameters in Bayesian estimation.

DEoptim_run_main
: If TRUE, run DEoptim in the main optimization.

deconst
: absolute value of lower and upper bound in DEoptim optimization.

numerical_deriv
: If TRUE, uses numerical derivative instead of the analytical.

best_method4start
: If TRUE, all optimizers are checked for starting values.

eqsys
: "sur" or "sem".

miterlim
: Number many iterations passed to maxLik function

opt_method
: optimization method for maxLik.

try_last_DEoptim
: If TRUE, in case of error in maxLik should DEoptim be run.

transform
: if TRUE, quantile transformation is applied to discrete equations.

MNtypef
: estimate "logit", or "dogit"

nmm_object
: nmm object created by nmm function.

object
: nmm object, for summary and nmm_sigma

type
: Type of standard errors c("robust", "clustered", "normal"), for summary

new_coef
: New coefficients that will be tested, for summary

...
: additional arguments affecting the summary produced, for summary

methodopt
: optimizer from maxLik package, for nmm_sigma

try_1good
: If TRUE, stops then first good values are found, for nmm_sigma

try_DEoptim
: If TRUE, uses DEoptim for optimization, then maxLik optimizers produce errors, for nmm_sigma

try_diff_method
: If TRUE, stops then first good values with Hessian check are found, for nmm_sigma

trace
: If TRUE, trace of DEoptim is printed, for nmm_sigma

Value

nmm
: returns nmm object with estimated parameters, functions, and data.

nmm_sigma
: returns estimated parameters, functions, data.

summary
: returns summary of nmm object.

Examples

```
# estimation of System of Nonlinear Equations based on example from 'systemfit'
library(systemfit)
data( ppine , package="systemfit")
hg.formula <- hg ~ exp( h0 + h1*log(tht) + h2*tht^2 + h3*elev)
dg.formula <- dg ~ exp( d0 + d1*log(dbh) + d2*hg + d3*cr)
```

```
labels <- list( "height.growth", "diameter.growth" )
model <- list( hg.formula, dg.formula )
start.values <- c(h0=-0.5, h1=0.5, h2=-0.001, h3=0.0001,
                  d0=-0.5, d1=0.009, d2=0.25, d3=0.005)
model.sur <- nlsystemfit( "SUR", model, start.values, data=ppine, eqnlabels=labels )
eq_c <- as.character(c(hg.formula, dg.formula))
parl <- c(paste0("h", 0:3),paste0("d", 0:3))
res <- nmm(ppine, eq_c=eq_c, par_c=parl, start_v = start.values,
eq_type = "cont", best_method = FALSE, numerical_deriv=TRUE)
summary(res)
res_sigma_cont <- nmm_sigma(res,estimate=TRUE) # Estimation of the Variance-Covariance matrix
summary(res_sigma_cont)

#example discrete choice
library(mlogit)
data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")
## a pure "conditional" model
mres <- summary(mlogit(mode ~ price + catch, data = Fish))
data <- prepare_data(Fish %>% data.frame %>% dplyr::select(-idx),
choice="alt", dummy="mode", PeID="chid", mode_spec_var = c("price", "catch"),
 type="long")
eq_d <- c("a1 + p1 * price_1 + p2 * catch_2", "a2 + p1 * price_2 + p2 * catch_2",
             "a3 + p1 * price_3 + p2 * catch_3", "a4 + p1 * price_4 + p2 * catch_4")
par_d <- c(paste0("a", 1:4), paste0("p", 1:2))
res <- nmm(data, eq_d=eq_d, par_d = par_d,  eq_type="disc", fixed_term=FALSE,
best_method=FALSE)
summary(res)

# joint estimation mockup example
data(dataM)
dataMp <- dataM %>% data.frame %>% prepare_data(. , choice="DR_Course",
PeID = "Student")
eq_c <- c("PlcmtScore ~ exp(a0 + a1 * PSATM + a2 * Rank + a3 * Size)",
"ACTM ~ exp(c0 + c1 * GPAadj)")
par_c <- c(paste0("a", 0:3), paste0("c", 0:1))
eq_d <- c("ASC1" ,
"ASC2 + b1_2 * SATM + b2_2 * PlcmtScore",
"ASC3 + b1_3 * SATM + b2_3 * PlcmtScore")
par_d <- c(paste0("ASC", 1:3), paste0("b", rep(1:2, rep(2,2)), "_", 2:3))

nmm_joint_res <- nmm(dataMp, eq_type = "joint", eq_d = eq_d,
par_d = par_d,  eq_c = eq_c, par_c = par_c,
start_v = c(a0=3.394, a1=0.001, a2=-0.001, a3=0, c0=3.583, c1=-0.008,
ASC2=-1.452, ASC3=3.047, b1_2=0.145, b1_3=0.102, b2_2=-0.133, b2_3=-0.168))
summary(nmm_joint_res)
```

`prepare_data`   `prepare_data` *prepare data for the estimation.*

Description

prepare_data prepare data for the estimation.

Usage

```
prepare_data(
  data,
  choice = "",
  dummy = "",
  PeID = "",
  WeID = "",
  type = "",
  mode_spec_var = "",
  avl = TRUE,
  chc = TRUE,
  wc = TRUE,
  wd = TRUE,
  nc = 0,
  weights = NULL,
  weight_paths = FALSE,
  weight_paths_cont = FALSE,
  mode_factors = NULL
)
```

Arguments

data	data.frame
choice	Name of variable with modes.
dummy	Name of variable indicating, if the mode was chosen.
PeID	Name of variable with individual identification numbers.
WeID	Name of variable with trip identification.
type	Type of data. If "long", then modifications are done.
mode_spec_var	Used if format "long", mode specific variables.
avl	if TRUE, includes dummies for mode availability.
chc	if TRUE, includes dummies for choice of mode.
wc	if TRUE, creates weights 1 for continuous equations.
wd	if TRUE, creates weights 1 for discrete equations.
nc	Integer, number of continuous equations.
weights	Data matrix with weights, column names have to be wc_i(continuous), wd_i(discrete).
weight_paths	if TRUE, weight according to number of trips per person, discrete part.
weight_paths_cont	if TRUE, weight continuous part.
mode_factors	if choice is not factor or numeric, this is important to supply.

Value

data.frame used for modeling.

Examples

```
data("TravelMode", package = "AER")
mode_spec_var <- c("wait", "vcost", "travel", "gcost")
res <- prepare_data(TravelMode, choice="mode", dummy="choice", PeID="individual", WeID="",
type="long", mode_spec_var =mode_spec_var, nc=3)
```

`pseudoR` *pseudo R^2*

Description

Calculates pseudo R^2 for discrete choice part.

Usage

```
pseudoR(
  x,
  which = c("all", "McFadden", "adjMcFadden", "Cox&Snell", "Nagelkerke"),
  only_total = FALSE
)
```

Arguments

`x`	Fitted `nmm` model.
`which`	which pseudo R^2 to calculate, options are: "all", "McFadden", "adjMcFadden", "Cox&Snell","Nagelkerke".
`only_total`	If `TRUE`, compute R^2 only for the whole sample.

Value

matrix with goodness of fit measures

Examples

```
library(mlogit)
data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")
## a pure "conditional" model
mres <- summary(mlogit(mode ~ price + catch, data = Fish))
data <- prepare_data(Fish %>% data.frame %>% dplyr::select(-idx),
choice="alt", dummy="mode", PeID="chid", mode_spec_var = c("price", "catch"),
type="long")
eq_d <- c("a1 + p1 * price_1 + p2 * catch_2", "a2 + p1 * price_2 + p2 * catch_2",
            "a3 + p1 * price_3 + p2 * catch_3", "a4 + p1 * price_4 + p2 * catch_4")
```

```
par_d <- c(paste0("a", 1:4), paste0("p", 1:2))
res <- nmm(data, eq_d=eq_d, par_d = par_d,  eq_type="disc", fixed_term=FALSE,
best_method=FALSE)
pseudoR(res, which = c("McFadden"))
ncf <- c(mres$coefficients)
names(ncf) <- par_d[-1]
mress <- in2nmm(res, new_coef =  ncf)
pseudoR(mress, which = c("McFadden"))
pseudoR(mress)
```

---

replace_par   replace_par *replaces text with other text.*

Description

replace_par replaces text with other text.

Usage

```
replace_par(iter, repdat, biogu)
```

Arguments

iter	Integer, which line of repdat is used, if 1 iteratively all will be replaced, if >1 only the i-th parameter.
repdat	data.frame with columns "old" and "new"
biogu	string which will be modified

Value

Modified string.

Examples

```
eq_c <- c("Tw ~ ((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) -(1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *
  (1 + (PH) + (tw)) *(-(PH) * (ta - Tc + 2) + (1 - (tw) * (ta - Tc + 2)) * (2/w -Ec/w))))/(2 *
   (1 + (PH) + (tw)))",
"Tf1 ~ (th1) * (ta - (((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) *(Ec/w - 2/w) - (1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta -Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *(1 + (PH) +
  (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 * (1 + (PH) +
   (tw)))) -Tc + 2) - 1",
"Ef1 ~ (ph1)/(PH) * (w * (((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) -
(1 + (PH))) + sqrt((((PH) + (tw)) *(ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 -4 *
 (1 + (PH) + (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 *
 (1 + (PH) + (tw)))) -Ec + 2) - 1")
parl <- c("tw","PH","th1","ph1")
parll <- paste0("par[", 1:length(parl), "]")
repdat <- data.frame(old=parl, new=parll)
replace_par(1, repdat, eq_c)
```

replace_par_wrap — replace_par_wrap *replace text with other text, wrapper of* replace_par.

Description

replace_par_wrap replace text with other text, wrapper of replace_par.

Usage

```
replace_par_wrap(repdat, obj)
```

Arguments

repdat	data.frame with columns "old" and "new"
obj	character string or vector of strings that will be modified.

Value

Modified string/vector.

Examples

```
eq_c <- c("Tw ~ ((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) -(1 + (PH))) +
sqrt((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *
 (1 + (PH) + (tw)) *(-(PH) * (ta - Tc + 2) + (1 - (tw) * (ta - Tc + 2)) * (2/w -Ec/w))))/(2 *
  (1 + (PH) + (tw)))",
"Tf1 ~ (th1) * (ta - (((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) *(Ec/w - 2/w) - (1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta -Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *(1 + (PH) +
  (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 * (1 + (PH) +
   (tw)))) -Tc + 2) - 1",
"Ef1 ~ (ph1)/(PH) * (w * (((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) -
(1 + (PH))) + sqrt((((PH) + (tw)) *(ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 -4 *
 (1 + (PH) + (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 *
 (1 + (PH) + (tw)))) -Ec + 2) - 1")
parl <- c("tw","PH","th1","ph1")
parll <- paste0("par[", 1:length(parl), "]")
repdat <- data.frame(old=parl, new=parll)
replace_par(2, repdat, eq_c)
replace_par_wrap(repdat, eq_c)
```

`stats_function` *Helper functions*

Description

Produce function that calculate estimates of endogenous variables from the continuous block and probabilities from discrete part.

Usage

```
stats_function(eq_c = NULL, eq_d = NULL, par_c = NULL, par_d = NULL, fixed = 0)
```

Arguments

eq_c	continuous equations
eq_d	discrete equations
par_c	parameters from cont. eq
par_d	parameters from disc. eq
fixed	index of fixed parameter

Value

Returns functions.

Examples

```
eq_d <- c("ASC1 * 1 + B11_dur * dur_1" , "ASC2 * 1 + B12_dur * dur_2",
"ASC3 * 1 + B13_dur * dur_3")
eq_c <- c("Tw ~ tw*w + ph1*Tc", "Tf1 ~ (1+w)^tw + ph1^3*Tc")
parl <- c("tw", "ph1")
par_d <- c(paste0("ASC", 1:3), paste0("B1", 1:3, "_dur"))
stfunc <- stats_function(eq_c, eq_d, parl,par_d, fixed=3)
data <- matrix(runif(1000, min=0.001, max=50), ncol=8)
data <- data.frame(data)
names(data) <- c("dur_1", "dur_2", "dur_3", "w", "Tc", "avl_1", "avl_2", "avl_3")
parv <- c(0.5, 1, 1.5, 2, 1, -0.3, 0.2, -0.8)
methodopt <- "BHHH"
separatenmm <- TRUE
env <- environment()
fnames <- c("prob_func", "cont_func")
sapply(1:(length(stfunc)-1), function(x)assign(fnames[x], stfunc[[x]], envir=env))
eval(parse(text=paste0("environment(", fnames[1:(length(stfunc)-1)], ") <- env")))
probs <- prob_func(parv)
apply(probs, 2, mean)
cont <- cont_func(parv)
apply(cont, 2, mean)
```

string2formula — `string2formula` *add square brackets to expressions. Reverse of formula2string. Convert par[2] <- par2, sigma[2] <- sigma_2_, sigma[2,3] <- sigma_2x2*

Description

`string2formula` add square brackets to expressions. Reverse of formula2string. Convert par[2] <- par2, sigma[2] <- sigma_2_, sigma[2,3] <- sigma_2x2

Usage

```
string2formula(x)
```

Arguments

`x`	String without square brackets.

Value

String with square brackets.

Examples

```
xx <- "par[1]*3+par[2]*par+rho1[1,2]+sigma1[2]"
xm <- formula2string(xx)
string2formula(xm)
```

wxMaxima — `wxMaxima` *does symbolic computation in 'Maxima' Requires installation of Maxima software.*

Description

`wxMaxima` does symbolic computation in 'Maxima' Requires installation of Maxima software.

Usage

```
wxMaxima(obj, out, tex = FALSE)
```

Arguments

`obj`	Lines without printed output, that will be evaluated in wxMaxima.
`out`	Lines with printed output, is end results of the function.
`tex`	if `TRUE` TeX expression of the printed output.

Value

List of character strings.

Examples

```
#components(determinant, Sigma inverse, argument for exponent) for joint normal distribution
eq_c <- c("Tw ~ ((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) -(1 + (PH))) +
sqrt((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 * (1 + (PH) +
(tw)) *(-(PH) * (ta - Tc + 2) + (1 - (tw) * (ta - Tc + 2)) * (2/w -Ec/w))))/(2 * (1 + (PH) +
(tw)))",
"Tf1 ~ (th1) * (ta - (((((PH) + (tw)) * (ta - Tc + 2) + (1 + (tw)) *(Ec/w - 2/w) - (1 + (PH))) +
 sqrt((((PH) + (tw)) * (ta -Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 - 4 *(1 + (PH) +
  (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 * (1 + (PH) +
   (tw)))) -Tc + 2) - 1",
"Ef1 ~ (ph1)/(PH) * (w * (((((PH) + (tw)) * (ta - Tc + 2) + (1 +(tw)) * (Ec/w - 2/w) -
(1 + (PH))) + sqrt((((PH) + (tw)) *(ta - Tc + 2) + (1 + (tw)) * (Ec/w - 2/w) - (1 + (PH)))^2 -4 *
 (1 + (PH) + (tw)) * (-(PH) * (ta - Tc + 2) + (1 - (tw) *(ta - Tc + 2)) * (2/w - Ec/w))))/(2 *
 (1 + (PH) + (tw)))) -Ec + 2) - 1")
parl <- c("tw","PH","th1","ph1")
para_cont <- get_par(parl, eq_c)
cheqs0 <- para_cont$cheqs0
npar <- get_npar(cheqs0)
neq <- length(cheqs0)
sdv <- rep(NA, neq)
mv <- rep(NA, neq)
sigma <- expand.grid(1:neq, 1:neq)%>%apply(., 1, function(x)paste0(x, collapse=','))%>%
paste0('sigma', '[', . ,']')%>%sort
sigma <- matrix(sigma, neq, neq, byrow = TRUE)
sigma[lower.tri(sigma)] <- sort(sigma[upper.tri(sigma)])
#create Y
y <- paste0('eps[', 1:neq,']')
ypy <- paste0('[', y, ']', collapse = ',')
ypy <- paste0('Y : matrix(', ypy, ')')
#sigma
spy <- paste0(sapply(1:neq, function(i) paste0('[', paste0(sigma[i,], collapse = ', '), ']')),
collapse = ',')
spy <- paste0('Sigma : matrix(', spy, ')')
detS <- 'dets : ratsimp(determinant(Sigma))'
invS <- 'invs : ratsimp(invert(Sigma))'
expt <- 'expt : ratsimp(transpose(Y) . invs . Y)'
obj <- c(ypy, spy, detS, invS, expt)
#grind function in Maxima returns an object that can be mathematically evaluated
out <- c('print(new)', 'grind(dets)',  'print(new)', 'grind(invs)', 'print(new)', 'grind(expt)')
## Not run:
rez <- wxMaxima(obj, out)

## End(Not run)
```

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